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2 PRELIMINARIES ON MATERIAL MODELLING

2.1 GENERAL DEFINITIONS OF STRESS AND STRAIN

A material model is a set of mathematical equations that describes the relationship between stress and strain. Material models are often expressed in a form in which infinitesimal increments of stress (often termed 'stress rates') are related to infinitesimal increments of strain (or 'strain rates'). All material models implemented in PLAXIS are based on a relationship between the effective stress rates, $\dot{\sigma}$, and the strain rates, $\dot{e}$. This relationship may be expressed in the form:

$$\dot{\sigma} = M \dot{e}$$  \hspace{1cm} (2.1)

There $M$ is a material stiffness matrix. Note that in this type of approach, pore-pressures are explicitly excluded from the stress-strain relationship.

In Eq. (2.1) the stress and strain rate tensors are written in vector notation, which involves, in general, six Cartesian components:

$$\dot{\sigma} = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx})^T$$  \hspace{1cm} (2.2a)

$$\dot{e} = (\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})^T$$  \hspace{1cm} (2.2b)

However, for plane strain and axisymmetric conditions, as modelled in PLAXIS, only four components are necessary, because $\gamma_{yz}$, $\gamma_{zx}$, $\sigma_{yz}$ and $\sigma_{zx}$ are zero. Positive normal stress components are considered to represent tension, whereas negative normal stress components indicate pressure (or compression). Similarly, positive normal strain components refer to extension, whereas negative normal strain components indicate compression.

![General three-dimensional coordinate system and sign convention](image)

Figure 2.1  General three-dimensional coordinate system and sign convention
Considering a small strain analysis, the strains are obtained from the spatial derivatives of the displacement components \(u_x\) and \(u_y\) by means of the following formulas:

\[
\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}
\]  

(2.3a)

\[
\varepsilon_{zz} = 0 \quad \text{(plane strain)}
\]  

(2.3b)

\[
\varepsilon_{zz} = \frac{1}{r} u_x \quad \text{(axisymmetry; } r = \text{ radius)}
\]  

(2.3c)

It is often useful to use principal stresses rather than Cartesian stress components when formulating certain material models. In the case of plane strain or axisymmetry, the principal stresses can be calculated from the Cartesian stresses by:

\[
\sigma_1' = \frac{1}{2}(\sigma_{xx}' + \sigma_{yy}') - \frac{1}{2}(\sigma_{xx}' - \sigma_{yy}')^2 + \sigma_{xy}'^2
\]  

(2.4a)

\[
\sigma_2' = \sigma_{zz}'
\]  

(2.4b)

\[
\sigma_3' = \frac{1}{2}(\sigma_{xx}' + \sigma_{yy}') + \frac{1}{2}(\sigma_{xx}' - \sigma_{yy}')^2 + \sigma_{xy}'^2
\]  

(2.4c)

In PLAXIS the principal stresses are arranged in algebraic order:

\[
\sigma_1' \leq \sigma_2' \leq \sigma_3'
\]  

(2.5)

Hence, \(\sigma_1'\) is the largest compressive stress. In this manual, models are often presented with reference to the principal stress space, as indicated in Fig. 2.2.

![Figure 2.2 Principal stress space](image)

Figure 2.2 Principal stress space
It is also useful to define invariants of stress, which are stress measures that are independent of the orientation of the coordinate system. Two useful stress invariants are:

\[ p' = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') \]  
\[ q = \frac{1}{2}( (\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2 ) \]

where \( p' \) is the isotropic stress, or mean effective stress, and \( q \) is the equivalent shear stress. Note that the convention adopted for \( p' \) is positive for compression in contrast to other stress measures. The equivalent shear stress, \( q \), has the important property that it reduces to \( q = |\sigma_1' - \sigma_3'| \) for triaxial stress states with \( \sigma_2' = \sigma_3' \).

In analogy to the mean stress, \( p' \), the volumetric strain, \( \varepsilon_v \), is defined as the sum of all principal strain components:

\[ \varepsilon_v = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) \]

The volumetric strain is defined as negative for compaction and as positive for dilatancy.

For elastoplastic models, as used in P\textsc{Laxis}, strains are decomposed into elastic and plastic components:

\[ \varepsilon = \varepsilon^e + \varepsilon^p \]

Throughout this manual, the superscript \( e \) will be used to denote elastic strains and the superscript \( p \) will be used to denote plastic strains.

### 2.2 ELASTIC STRAINS

The simplest material model in P\textsc{Laxis} is Hooke's law for isotropic linear elastic behaviour (Linear Elastic model), which is given by the equation:

\[
\begin{bmatrix}
\sigma_{xx}' \\
\sigma_{yy}' \\
\sigma_{zz}' \\
\sigma_{xy}'
\end{bmatrix} = \frac{E'}{(1-2v')(1+v')} \begin{bmatrix}
1-v' & v' & v' & 0 \\
v' & 1-v' & v' & 0 \\
v' & v' & 1-v' & 0 \\
0 & 0 & 0 & \%e - v'
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx}' \\
\varepsilon_{yy}' \\
\varepsilon_{zz}' \\
\varepsilon_{xy}'
\end{bmatrix}
\]

\[(2.9)\]
The elastic material stiffness matrix is often denoted as $D$. Two parameters are used in this model, the effective Young's modulus, $E'$, and the effective Poisson's ratio, $\nu'$. In the remaining part of this manual effective parameters are denoted without dash ('), unless a different meaning is explicitly stated. The symbols $E$ and $\nu$ are sometimes used in this manual in combination with the subscript $ur$ to emphasize that the parameter is explicitly meant for unloading and reloading. A stiffness modulus may also be indicated with the subscript $ref$ to emphasize that it refers to a particular reference level ($y_{ref}$) (see further).

The relationship between Young's modulus $E$ and other stiffness moduli, such as the shear modulus $G$, the bulk modulus $K$, and the oedometer modulus $E_{oed}$, is given by:

$$G = \frac{E}{2(1+\nu)} \quad K = \frac{E}{3(1-2\nu)} \quad E_{oed} = \frac{(1-\nu)E}{(1-2\nu)(1+\nu)}$$

(2.10)

During the input of material parameters for the Linear Elastic model or the Mohr-Coulomb model the values of $G$ and $E_{oed}$ are presented as auxiliary parameters (alternatives), calculated from Eq. (2.10). Note that the alternatives are influenced by the input values of $E$ and $\nu$. Entering a particular value for one of the alternatives $G$ or $E_{oed}$ results in a change of the $E$ modulus.

It is possible for the Linear Elastic model to specify a stiffness that varies linearly with depth. This can be done by entering the advanced parameters window using the Advanced button, as shown in figure 2.3. Here one may enter a value for $E_{increment}$ which is the increment of stiffness per unit of depth, as indicated in Fig. 2.4.
Together with the input of $E_{\text{increment}}$ the input of $y_{\text{ref}}$ becomes relevant. Above $y_{\text{ref}}$ the stiffness is equal to $E_{\text{ref}}$. Below the stiffness is given by:

$$E_{\text{actual}} = E_{\text{ref}} + (y_{\text{ref}} - y) E_{\text{increment}} \quad (y < y_{\text{ref}}) \quad (2.11)$$

Figure 2.4 Advanced parameter window

The Linear Elastic model is usually inappropriate to model the highly non-linear behaviour of soil, but it is of interest to simulate structural behaviour, such as thick concrete walls or plates, for which strength properties are usually very high compared with those of soil. For these applications, the Linear Elastic model will often be selected together with Non-porous type of material behaviour in order to exclude pore pressures from these structural elements.

### 2.3 UNDRAINED ANALYSIS WITH EFFECTIVE PARAMETERS

In PLAXIS it is possible to specify undrained behaviour in an effective stress analysis using effective model parameters. This is achieved by identifying the Type of material behaviour (Material type) of a soil layer as Undrained. In this Section, it is explained how PLAXIS deals with this special option.

The presence of pore pressures in a soil body, usually caused by water, contributes to the total stress level. According to Terzaghi’s principle, total stresses $\sigma$ can be divided into effective stresses $\sigma'$ and pore pressures $\sigma_w$:

$$\begin{align*}
\sigma_{xx} &= \sigma'_{xx} + \sigma_w \\
\sigma_{yy} &= \sigma'_{yy} + \sigma_w \\
\sigma_{zz} &= \sigma'_{zz} + \sigma_w \\
\sigma_{xy} &= \sigma'_{xy}
\end{align*} \quad (2.12)$$

Note that, similar to the total and the effective stress components, $\sigma_w$ is considered negative for pressure.
A further distinction is made between steady state pore stress, \( p_{\text{steady}} \), and excess pore stress, \( p_{\text{excess}} \):

\[
\sigma_w = p_{\text{steady}} + p_{\text{excess}}
\]  

(2.13)

Steady state pore pressures are considered to be input data, either generated on the basis of phreatic lines or by means of a groundwater flow calculation. This generation of steady state pore pressures is extensively discussed in Section 3.8 of the Reference Manual. Excess pore pressures are generated during plastic calculations for the case of undrained material behaviour. The behaviour of the system while the pore pressures dissipate with time may be studied using a consolidation analysis. Undrained material behaviour, and the corresponding calculation of excess pore pressures, are described below.

Since the time derivative of the steady state component equals zero, it follows:

\[
\sigma_w = \dot{p}_{\text{excess}}
\]  

(2.14)

Hooke's law (2.9) can be inverted to obtain:

\[
\begin{bmatrix}
\dot{e}_{xx}' \\
\dot{e}_{yy}' \\
\dot{e}_{zz}' \\
\dot{e}_{xy}'
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
1 & -\nu & -\nu & 0 \\
-\nu & 1 & -\nu & 0 \\
-\nu & -\nu & 1 & 0 \\
0 & 0 & 0 & 2 + 2\nu
\end{bmatrix} \begin{bmatrix}
\sigma_{xx}' \\
\sigma_{yy}' \\
\sigma_{zz}' \\
\sigma_{xy}'
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
1 & -\nu & -\nu & 0 \\
-\nu & 1 & -\nu & 0 \\
-\nu & -\nu & 1 & 0 \\
0 & 0 & 0 & 2 + 2\nu
\end{bmatrix} \begin{bmatrix}
\sigma_{xx} - \sigma_w \\
\sigma_{yy} - \sigma_w \\
\sigma_{zz} - \sigma_w \\
\sigma_{xy}
\end{bmatrix} 
\]  

(2.15)

Considering slightly compressible water, the pore pressure rate is written as:

\[
\sigma_w = \frac{K_w}{n} (\dot{e}_{xx}' + \dot{e}_{yy}' + \dot{e}_{zz}')
\]  

(2.16)

in which \( K_w \) is the bulk modulus of the water and \( n \) is the soil porosity.

The inverted form of Hooke's law may be written in terms of the total stress rates and the undrained parameters \( E_u \) and \( \nu_u \):

\[
\begin{bmatrix}
\dot{e}_{xx}' \\
\dot{e}_{yy}' \\
\dot{e}_{zz}' \\
\dot{e}_{xy}'
\end{bmatrix} = \frac{1}{E_u} \begin{bmatrix}
1 & -\nu_u & -\nu_u & 0 \\
-\nu_u & 1 & -\nu_u & 0 \\
-\nu_u & -\nu_u & 1 & 0 \\
0 & 0 & 0 & 2 + 2\nu_u
\end{bmatrix} \begin{bmatrix}
\sigma_{xx}' \\
\sigma_{yy}' \\
\sigma_{zz}' \\
\sigma_{xy}'
\end{bmatrix} = \frac{1}{E_u} \begin{bmatrix}
1 & -\nu_u & -\nu_u & 0 \\
-\nu_u & 1 & -\nu_u & 0 \\
-\nu_u & -\nu_u & 1 & 0 \\
0 & 0 & 0 & 2 + 2\nu_u
\end{bmatrix} \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xy}
\end{bmatrix} 
\]  

(2.17)
where:

\[
E_u = 2G(1 + \nu_u) \quad \mu_u = \frac{\nu + \mu (1+\nu)}{1 + 2\mu (1+\nu)} \quad (2.18)
\]

\[
\mu = \frac{1}{3} \frac{K_w}{n K} \quad K = \frac{E}{3(1-2\nu)} \quad (2.19)
\]

Hence, the special option for undrained behaviour in PLAXIS is such that the effective parameters \(G\) and \(\nu\) are transferred into undrained parameters \(E_u\) and \(\nu_u\) according to Eq. (2.18) and (2.19). Note that the index \(u\) is used to indicate auxiliary parameters for undrained soil. Hence, \(E_u\) and \(\nu_u\) should not be confused with \(E_{ur}\) and \(\nu_{ur}\) as used to denote unloading / reloading.

Fully incompressible behaviour is obtained for \(\nu_u = 0.5\). However, taking \(\nu_u = 0.5\) leads to singularity of the stiffness matrix. In fact, water is not fully incompressible, but a realistic bulk modulus for water is very large. In order to avoid numerical problems caused by an extremely low compressibility, \(\nu_u\) is taken as 0.495, which makes the undrained soil body slightly compressible. In order to ensure realistic computational results, the bulk modulus of the water must be high compared with the bulk modulus of the soil skeleton, i.e. \(K_w >> n K\). This condition is sufficiently ensured by requiring \(\nu < 0.35\). Users will get a warning as soon as larger Poisson's ratios are used in combination with undrained material behaviour.

Consequently, for undrained material behaviour a bulk modulus for water is automatically added to the stiffness matrix. The value of the bulk modulus is given by:

\[
\frac{K_w}{n} = \frac{3(\nu_u - \nu)}{(1 - 2\nu_u)(1+\nu)} K = 300 \frac{0.495 - \nu}{1 + \nu} K > 30 K \quad (2.20)
\]

at least for \(\nu < 0.35\).

The rate of excess pore pressure is calculated from the (small) volumetric strain rate, according to:

\[
\sigma_w = \frac{K_w}{n} \dot{\varepsilon}_v \quad (2.21)
\]

The type of elements used in PLAXIS are sufficiently adequate to avoid mesh locking effects for nearly incompressible materials.

This special option to model undrained material behaviour on the basis of effective model parameters is available for all material models in PLAXIS. This enables undrained calculations to be executed with effective input parameters, with explicit distinction between effective stresses and (excess) pore pressures.
Such an analysis requires effective soil parameters and is therefore highly convenient when such parameters are available. For soft soil projects, accurate data on effective parameters may not always be available. Instead, in situ tests and laboratory tests may have been performed to obtain undrained soil parameters. In such situations measured undrained Young's moduli can be easily converted into effective Young's moduli by:

\[ E = \frac{2}{3} \left( 1 + \nu \right) E_u \]  

(2.22)

Undrained shear strengths, however, cannot easily be used to determine the effective strength parameters \( \phi \) and \( c \). For such projects PLAXIS offers the possibility of an undrained analysis with direct input of the undrained shear strength \( (c_u \text{ or } s_u) \) and \( \phi = \phi_u = 0^\circ \). This option is only available for the Mohr-Coulomb model and the Hardening-Soil model, but not for the Soft Soil (Creep) models. Note that whenever the Material type parameter is set to Undrained, effective values must be entered for the elastic parameters \( E \) and \( \nu \)!

2.4 UNDRAINED ANALYSIS WITH UNDRAINED PARAMETERS

If, for any reason, it is desired not to use the Undrained option in PLAXIS to perform an undrained analysis, one may simulate undrained behaviour by selecting the Non-porous option and directly entering undrained elastic properties \( E = E_u \) and \( \nu = \nu_u = 0.495 \) in combination with the undrained strength properties \( c = c_u \) and \( \phi = \phi_u = 0^\circ \). In this case a total stress analysis is performed without distinction between effective stresses and pore pressures. Hence, all tabulated output referring to effective stresses should now be interpreted as total stresses and all pore pressures are equal to zero. In graphical output of stresses the stresses in Non-porous clusters are not plotted. If one does want graphical output of stresses one should select Drained instead of Non-porous for the type of material behaviour and make sure that no pore pressures are generated in these clusters.

Note that this type of approach is not possible when using the Soft Soil (Creep) models. In general, an effective stress analysis using the Undrained option in PLAXIS to simulate undrained behaviour is preferable over a total stress analysis.

2.5 THE INITIAL PRECONSOLIDATION STRESS IN ADVANCED MODELS

When using advanced models in PLAXIS an initial preconsolidation stress has to be determined. In the engineering practice it is common to use a vertical preconsolidation stress, \( \sigma_p \), but PLAXIS needs an equivalent isotropic preconsolidation stress, \( p_{eq}^{cap} \), to determine the initial position of a cap-type yield surface. If a material is overconsolidated, information is required about the Over-Consolidation Ratio (OCR),
i.e. the ratio of the greatest vertical stress previously reached, \( \sigma_p \) (see Fig. 2.5), and the in-situ effective vertical stress, \( \sigma_{yy}^0 \).

\[
OCR = \frac{\sigma_p}{\sigma_{yy}^0} \tag{2.23}
\]

It is also possible to specify the initial stress state using the Pre-Overburden Pressure (POP) as an alternative to prescribing the overconsolidation ratio. The Pre-Overburden Pressure is defined by:

\[
POP = |\sigma_p - \sigma_{yy}^0| \tag{2.24}
\]

These two ways of specifying the vertical preconsolidation stress are illustrated in Fig. 2.5.

![Figure 2.5 Illustration of vertical preconsolidation stress in relation to the in-situ vertical stress. 2.5a. Using OCR; 2.5b. Using POP](image)

The pre-consolidation stress \( \sigma_p \) is used to compute \( \sigma_{p^e} \) which determines the initial position of a cap-type yield surface in the advanced soil models. The calculation of \( \sigma_{p^e} \) is based on the stress state:

\[
\sigma_1' = \sigma_p \quad \text{and} \quad \sigma_2' = \sigma_3' = K_{NC}^0 \sigma_p \tag{2.25}
\]

Where \( K_{NC}^0 \) is the \( K_0 \)-value associated with normally consolidated states of stress. Both for the Hardening-Soil model and the Soft-Soil model default parameter settings are such that we have the Jaky formula \( K_{NC}^0 = 1 - \sin \phi \). For the Soft-Soil-Creep model, the default setting is slightly different, but differences with the Jaky correlation are modest.
2.6 ON THE INITIAL STRESSES

In overconsolidated soils the coefficient of lateral earth pressure is larger than for normally consolidated soils. This effect is automatically taken into account for advanced soil models when generating the initial stresses using the $K_0$ procedure. The procedure that is followed here is described below.

Consider a one-dimensional compression test, preloaded to $\sigma_{yy} = \sigma_p$ and subsequently unloaded to $\sigma_{yy}' = \sigma_{yy}^0$. During unloading the sample behaves elastically and the incremental stress ratio is, according to Hooke's law, given by (see Fig. 2.6):

$$\frac{\Delta \sigma_{xx}'}{\Delta \sigma_{yy}'} = \frac{K_0^{NC} \sigma_p \cdot \sigma_{xx}^0}{\sigma_p \cdot \sigma_{yy}^0} = \frac{K_0^{NC} \sigma_{yy}' - \sigma_{yy}^0}{(OCR - 1) \sigma_{yy}^0} = \frac{\nu_{ur}}{1 - \nu_{ur}} \quad (2.26)$$

where $K_0^{nc}$ is the stress ratio in the normally consolidated state. Hence, the stress ratio of the overconsolidated soil sample is given by:

$$\frac{\sigma_{xx}'}{\sigma_{yy}'} = K_0^{NC} OCR \cdot \frac{\nu_{ur}}{1 - \nu_{ur}} \quad (OCR - 1) \quad (2.27)$$

The use of a small Poisson's ratio, as discussed previously, will lead to a relatively large ratio of lateral stress and vertical stress, as generally observed in overconsolidated soils. Note that Eq. (2.27) is only valid in the elastic domain, because the formula was derived from Hooke's law of elasticity. If a soil sample is unloaded by a large amount, resulting in a high degree of overconsolidation, the stress ratio will be limited by the Mohr-Coulomb failure condition.

![Figure 2.6 Overconsolidated stress state obtained from primary loading and subsequent unloading](image)