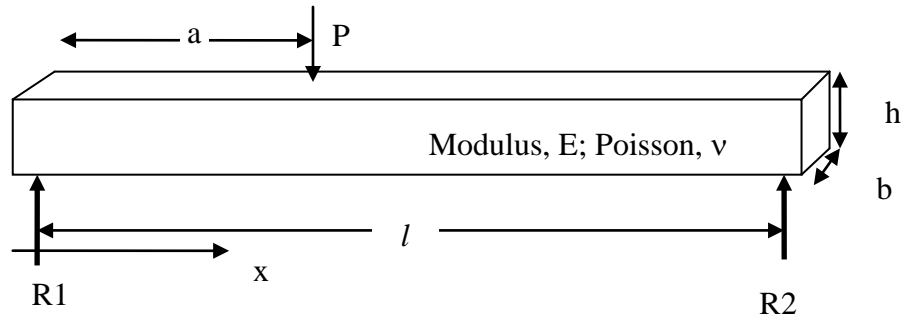


ECIV 201 Simple Beam Parametric Study

This example is to illustrate a method of analysis for a system (a beam in this case). The system can be quite complicated, but in this instance it is a simply-supported beam with a single, moveable concentrated load.



Given the diagram above, one can define several quantities:

- P = Load (F)
- R1 = Reaction at left (F)
- R2 = Reaction at right (F)
- E = Young's Modulus of beam material ($F L^{-2}$)
- ν = Poisson's Ratio of beam material ()
- l = Length of Beam (L)
- h = Height of beam (L)
- b = Width of beam (L)
- a = Position of load P (L)
- x = Location anywhere along beam (L)

Based on our vast and weighty knowledge of statics and mechanics, we can define the following relationships

$$R1 + R2 = P$$

$$R1(l) = P(l-a) \quad R1 = P(l-a)/l$$

$$R2 = P - P(l-a)/l$$

Moment at location x,

$$M = R2(l-x) - P\langle a-x \rangle \quad \text{if } x > a, \langle a-x \rangle = 0$$

Stress and strain due to bending at location x,

$$\sigma = M(y)/I \quad \text{where } y = \text{distance from neutral axis (center)}, \quad I = bh^3/12$$

$$\epsilon_x = 1/E (\sigma_x - 2\sigma_y \nu)$$

Deflection of neutral axis

$$\delta = \frac{-P}{6EI} \left[\frac{(l-a)(x)}{l} (l^2 - (l-a)^2 - x^2) + \langle x-a \rangle^3 \right]$$

Now, given a reasonable range of values, one can proceed with the parameter study. Note that some of these numbers are limited by physical properties (Young's Modulus, Poisson's Ratio), some by practical limitations (lumber is rarely longer than 20 feet), or by problem constraints ($a < l$).

So, what will be our variable of interest? Maybe stress, maybe strain, maybe deflection. Let's choose deflection as our variable of interest. I use the term variable here because I am not controlling it; the equation will dictate its behavior. What I will control are all the other numbers (my *parameters*).

Typical ranges of values for our study are:

$l = 8$ to 16 ft.

$b = 0.2$ to 0.4 ft

$h = 0.3$ to 1.0 ft

$E = 1 \times 10^6$ to 30×10^6 psi (convert to lb/ft²)

$\nu = 0.2$ to 0.45

$P = 20$ to 200 lb

Now it just becomes a matter of organization to bring your study to life.

1. Determine useful illustrations of behavior (what do you want to show your reader)
2. Compute them in a systemic manner (how can you organize your computations)
3. Present results in a clear manner (how do you make your graphs)
4. Determine and highlight critical values (you are trying to make a point, make it)

The types of information one may want to know are best asked as questions such as:

1. What is the influence of load location on deflection?
2. Where does maximum deflection occur for a given load application?
3. What are the influences of cross-sectional properties?
4. How do I have to change the beam's properties to double the load but maintain the same deflection?

Assignment

Part 1

For starters, plot the deflection of the beam (δ) for different locations (a) of load (P). Use the following values:

$$\begin{aligned} P &= 100 \text{ lbs.} & b &= 0.25 \text{ ft} \\ l &= 12 \text{ ft} & E &= 1.0 \times 10^6 \text{ psi} \\ h &= 0.75 \text{ ft} \end{aligned}$$

Plot the deflections at $x=0, 1, 2, \dots, 12$ ft

Apply the load at $a=6, 5, 4, \dots, 0$ ft

Where is the location of maximum deflection? Does the deflected shape change?

Part 2

For your next trick, study the effect of height of beam, h on the maximum fiber stress σ . Set $b=1.75$ inches (how many feet?) and vary h from 1.0 inches to 12 inches. Keep the beam loaded at the middle.

Part 3

Choose another parameter to study, ask your questions and find your answers.