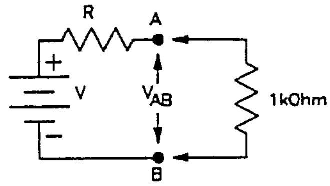
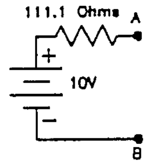
Fundamentals of Engineering Exam
Electrical Review - Sample Problems

DC Circuits

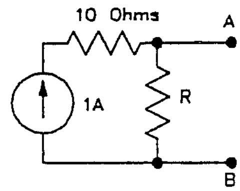
1. A power supply is modeled as a voltage V and internal resistance R with output terminals A and B. It has an open circuit voltage $V_{AB(OC)}$ of 10 volts. When a 1kΩ external load resistor is connected, V_{AB} decreases to 9 volts. Find the Thevenin's equivalent circuit of the power supply.



Solution: Since $V_{AB(OC)}$=10V, V_{TH}=V_{OC}=10V. When the 1kΩ resistor is connected, the voltage across it will be 9V. Therefore, the current will be I = V/R = 9/1000 = 9mA. Also, when this occurs, the voltage across the internal resistor R is 10V-9V = 1V. Therefore, the resistor R is R = V/I = 1/0.009 = 111.1Ω. The solution is shown to the right.

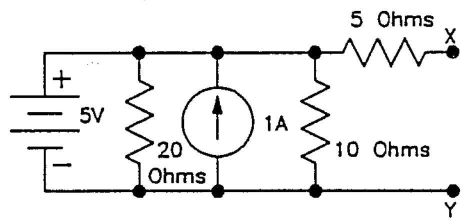


2. What value of R will make V_{AB} = 1V ?



Solution: First, note that the 10Ω resistor is simply there to add confusion to the problem, e.i. the 10Ω resistor could be any value and the current through R would still be 1 ampere. Therefore the answer is R = V/I = 1/1 = 1Ω

3. Find the equivalent resistance R_{eq} between terminals X,Y.

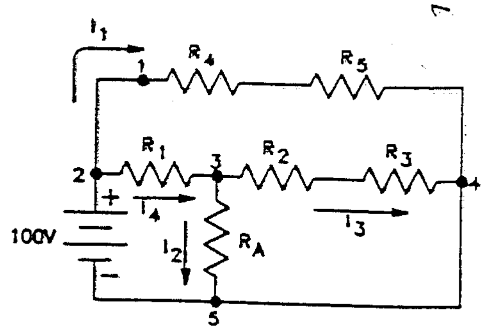


Solution: To find equivalent resistance, remove all sources by replacing each voltage source with a short circuit and each current source with an open circuit. In this circuit, when we replace the 5V voltage source with a short, it connects the left side of the 5Ω resistor to terminal Y and bypasses (shorts out) all the other components. Therefore, the equivalent resistance is R_{eq} = 5Ω. The 20Ω resistor, 1A current source, and 10Ω resistor have no bearing on this problem.

4. For the circuit in Problem 3, what is the open circuit voltage $V_{XY(OC)}$?

Solution: Since the 5V voltage source, the 20Ω resistor, the 1A current source, and the 10Ω resistor are all in parallel, we can rearrange their positions as needed. Therefore, swap the positions of the 10Ω resistor and the 5V voltage source. Since no current is flowing through the 5Ω resistor, the voltage drop across the resistor is zero. Therefore, the voltage $V_{XY(OC)}$ is equal to the potential of the voltage source, which is 5V. Again, the 20Ω resistor, 1A current source, and 10Ω resistor have no bearing on this problem. Also, the value of the 5Ω resistor does not matter.

5. For this problem $R_1 = R_2 = R_3 = R_4 = R_5 = 20\Omega$, R_A is unknown, $V_{25} = 100V$, $V_{35} = 27.57V$, and $V_{45} = 0V$.



a) Find I_1

Solution: I_1 is the current through R_4 and R_5. The left side of R_4 is connected to the + side of the battery, and the right side of R_5 is connected to the - side of the battery. Therefore, $V_{R4+R5} = 100V$ and

$$I_1 = \frac{V_{R4+R5}}{R_4+R_5} = \frac{100}{20+20} = 2.5 \text{ Amperes}$$

b) Find R_A

Solution: We know that the voltage drop across R_A is the same as $V_{35} = 27.57V$, but we must also know I_2 in order to calculate R_A. We can find the voltage drop on R_1 which is $V_{23} = V_{25} - V_{35}$ (KVL). Therefore I_4 is

$$I_4 = \frac{V}{R} = \frac{V_{25}-V_{35}}{R_1} = \frac{100-27.57}{20} = 3.62 \text{ Amperes}$$

We can find I_3 the same way:

$$I_3 = \frac{V}{R} = \frac{V_{35}-V_{45}}{R_2+R_3} = \frac{27.57-0}{20+20} = 0.69 \text{ Ampere}$$

Using KCL at node 3, we get $-I_4+I_2+I_3 = 0$, or

$$I_2 = I_4 - I_3 = 3.62 - 0.69 = 2.93 \text{ Amperes}$$

Now we find R_A which is

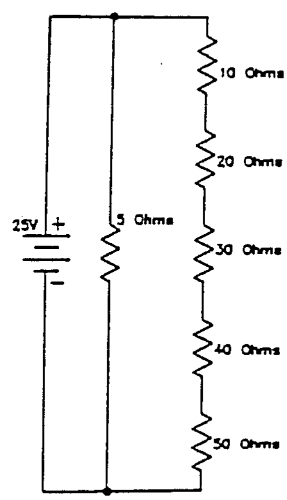
$$R_A = \frac{V}{I} = \frac{V_{35}}{I_2} = \frac{27.57}{2.93} = 9.4\Omega$$

c) Find the power dissipated by R_5.

Solution: We found I_1 in part a). Use it to calculate the power dissipated by R_5:

$$P_{R_5} = I_1^2 R_5 = 2.5^2 \times 20 = 125 \text{ Watts}$$

6. Find the voltage drop across the 30Ω resistor.

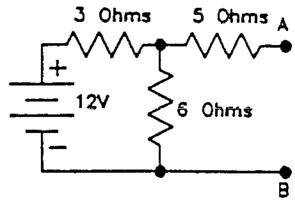


Solution: Use the voltage divider rule.

$$V_{30\Omega} = V_{IN}\left(\frac{30\Omega}{R_{TOTAL}}\right) = 25 \times \left(\frac{30}{10+20+30+40+50}\right) = 5V$$

The 5Ω resistor has no bearing on this problem.

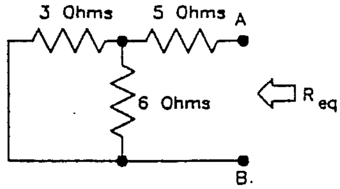
7. Find the Norton equivalent of this circuit as "seen" from terminals AB.



Solution: First, find $V_{TH} = V_{OC} = V_{AB(OC)}$. With AB open circuited, no voltage will be dropped across the 5Ω resistor, and V_{AB} will equal the voltage drop on the 6Ω resistor. Therefore, by the voltage divider rule, V_{TH} is

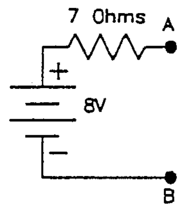
$$V_{TH} = 12\left(\frac{6}{3+6}\right) = 8\text{V}$$

Now short out the 12V battery and find $R_{TH} = R_{eq}$ at terminals AB.

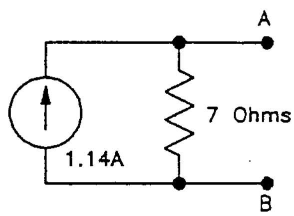


$$R_{eq} = 5\Omega + (3\Omega \parallel 6\Omega) = 5 + \left(\frac{3\times 6}{3+6}\right) = 7\Omega$$

Therefore, the Thevenin's equivalent circuit is an 8V voltage source in series with a 7Ω resistor as shown to the right.



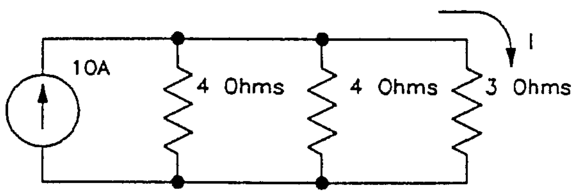
Now find the Norton's equivalent circuit by doing a source transformation on the Thevenin's circuit.



$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{8}{7} = 1.14 \text{ Amperes}$$

$$R_N = R_{TH} = 7\Omega$$

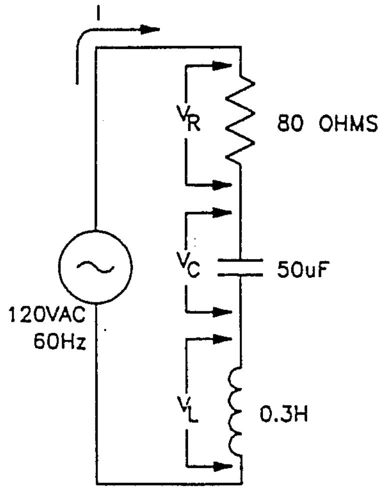
8. Find the current I through the 3Ω resistor.



Solution: Use the current divider rule. First combine the two 4Ω resistors to make a single 2Ω resistor. Now we can find I whch is

$$I = I_{IN}\left(\frac{2\Omega}{2\Omega + 3\Omega}\right) = 10\left(\frac{2}{5}\right) = 4 \text{ Amperes}$$

AC Circuits



9. For this circuit, find **a)** the load impedance "seen" by the voltage source, **b)** the current I, **c)** voltages V_R, V_C, V_L, and **d)** the power dissipated by the circuit and the power factor.

Solutions:

a) First, find the impedance of each component.

$$X_L = 2\pi fL = 2\pi(60)(0.3) = 113\Omega\angle{+90^\circ} = 0 + j113$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60)(50\times10^{-6})} = 53\Omega\angle{-90^\circ} = 0 - j53$$

$$R = 80\Omega\angle 0^\circ = 80 + j0$$

Now find the total impedance.

$$Z = X_L + X_C + R = (0 + j113) + (0 - j53) + (80 + j0)$$
$$= 80 + j60 = 100\Omega\angle{+36.87^\circ}$$

b) Using ohm's law, the current is

$$I = \frac{V}{Z} = \frac{120\angle 0^\circ}{100\angle{+36.87}} = 1.2\angle{-36.87^\circ}\ amperes$$

c) Again, using ohm's law the voltage drops are

$$V_R = IR = (1.2\angle{-36.87^\circ})(80\angle 0^\circ) = 96\angle{-36.87^\circ}\ volts$$
$$V_C = IX_C = (1.2\angle{-36.87^\circ})(53\angle{-90^\circ}) = 63.6\angle{-126.87^\circ}\ volts$$
$$V_L = IX_L = (1.2\angle{-36.87^\circ})(113\angle{+90^\circ}) = 135.6\angle{+53.13^\circ}\ volts$$

We can check our answer at this point. By KVL, the vector sum of these three voltages must total $120V\angle 0^\circ$. This is done by converting the three voltages from polar to rectangular form and individually summing the real and imaginary parts.

d) Since reactive components do not dissipate power, the only power dissipated is in the resistor. This power is a scalar value and is

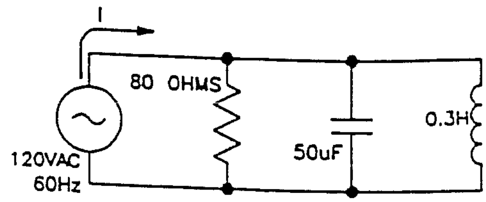
$$P_{TOTAL} = P_R = I^2 R = (1.2^2)(80) = 115.2\ watts$$

e) Power factor is calculated in various ways. In this case, the simplest way is

$$Power\ Factor = \frac{Real\ Power}{Apparent\ Power} = \frac{P_r}{P_a} = \frac{I^2 R}{VI} = \frac{(1.2^2)(80)}{(120)(1.2)} = 0.8\ lagging$$

We can determine that the power factor is lagging by observing the angle of the current with respect to the angle of the applied voltage. The current is lagging the voltage (it has a negative phase angle with respect to the angle of the voltage source).

10. Find the total current supplied by the source.



Solution: The component impedances were calculated in problem 9. Using the polar forms of the impedances, and the parallel impedance equation we get the overall impedance.

$$Z = \frac{1}{\frac{1}{R} + \frac{1}{X_C} + \frac{1}{X_L}} = \frac{1}{\frac{1}{80\angle 0^\circ} + \frac{1}{53\angle -90^\circ} + \frac{1}{113\angle +90^\circ}}$$

$$= \frac{1}{(0.0125\angle 0^\circ) + (0.0188\angle +90^\circ) + (0.0088\angle -90^\circ)}$$

$$= \frac{1}{(0.0125\angle 0^\circ) + (0.01\angle +90^\circ)}$$

$$= \frac{1}{(0.0125 + j0) + (0 + j0.01)} = \frac{1}{0.0125 + j0.01} = \frac{1}{0.016\angle +38.66^\circ}$$

$$= 62.47\angle -38.66^\circ\ \Omega$$

Now we can use ohm's law to find the current.

$$I = \frac{V}{Z} = \frac{120\angle 0^\circ}{62.47\angle -38.66^\circ} = 1.92\angle 38.66^\circ\ amperes$$

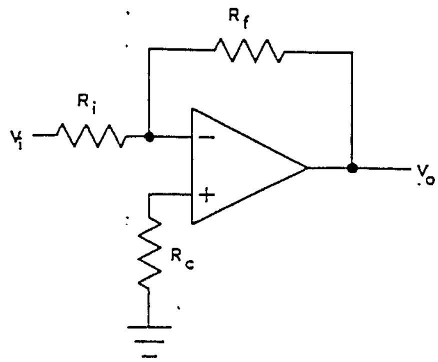
In this case the current (and power factor) will be leading because the phase angle of the current is positive with respect to the voltage.

OPERATIONAL AMPLIFIERS

General Information:

Typical "op amp" circuits fall into two catagories: inverting and non-inverting.

Inverting Amplifier



The voltage gain A_V of the circuit is

$$A_V = -\frac{R_f}{R_i}$$

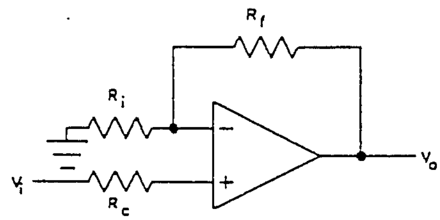
and the relationship between the input voltage V_i and the output voltage V_o is

$$V_o = A_V V_i = \left(-\frac{R_f}{R_i}\right) V_i$$

Resistor R_c has no bearing on the voltage gain of the circuit and is used for temperature stability of the circuit. If asked to calculate the value of R_c, it is

$$R_c = \frac{1}{\frac{1}{R_f} + \frac{1}{R_i}}$$

Non-Inverting Amplifier



The voltage gain A_V of the circuit is

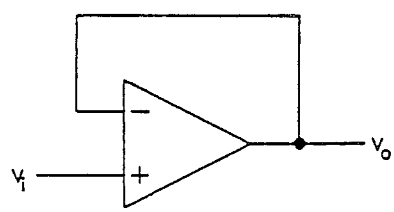
$$A_V = \frac{R_i + R_f}{R_i}$$

and the relationship between the input voltage V_i and the output voltage V_o is

$$V_o = A_V V_i = \left(\frac{R_i + R_f}{R_i}\right) V_i$$

Again, resistor R_c has no bearing on the voltage gain of the circuit and the value of R_c is

$$R_c = \frac{1}{\frac{1}{R_f} + \frac{1}{R_i}}$$



There is a special case of the non-inverting amplifier where $R_f = R_c = 0\Omega$ and $R_i = \infty$. This is called a "voltage follower". For this amplifier, the voltage gain is

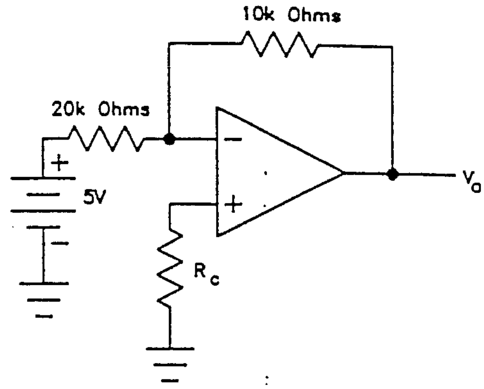
$$A_V = +1$$

and the relationship between the input voltage V_i and the output voltage V_o is

$$V_o = V_i$$

Sample Problems:

11. For this amplifier, find a) the voltage gain A_V, b) the output voltage V_o, c) and the value of R_c.



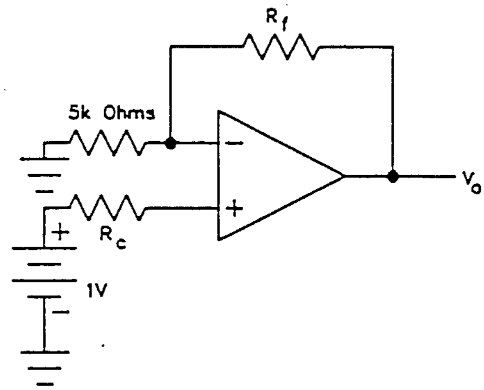
Solution:

$$A_V = -\frac{R_f}{R_i} = -\frac{10k}{20k} = -0.5$$

$$V_o = A_V V_i = (-0.5)(5V) = -2.5V$$

$$R_c = \frac{1}{\frac{1}{R_f} + \frac{1}{R_i}} = \frac{1}{\frac{1}{10k} + \frac{1}{20k}} = 6.6\overline{6}\ k\Omega$$

12. What value of R_f will make V_o = 5 volts?



Solution:
First, we must calculate A_V. To do this, we take the V_o equation

$$V_o = A_V V_i$$

and solve for A_V.

$$A_V = \frac{V_o}{V_i} = \frac{5V}{1V} = 5$$

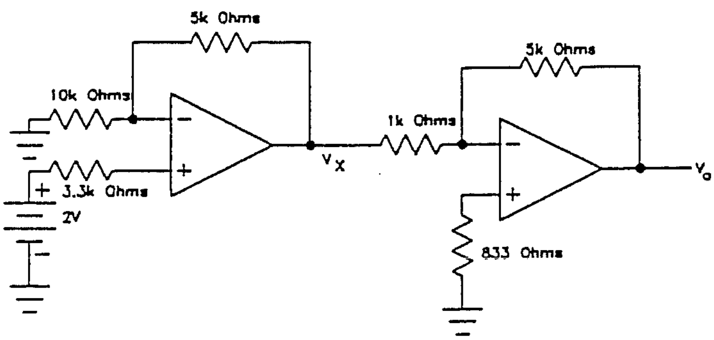
Now find R_f by choosing the A_V equation

$$A_V = \frac{R_i + R_f}{R_i}$$

and solving for R_f to get

$$R_f = A_V R_i - R_i = R_i(A_V - 1) = 5k\Omega(5 - 1) = 20k\Omega$$

13. Find the output voltage V_o.



Solution:
This is called a "two stage amplifier". The first stage (on the left) is a non-inverting amplifier with an input voltage of 2 volts and an output V_X. The second stage is an inverting amplifier with an input V_X and an output V_o. First, we find the 1st stage output voltage V_X.

$$V_X = \left(\frac{R_i + R_f}{R_i}\right) V_i = \left(\frac{10k + 5k}{10k}\right) 2V = 3\ volts$$

Now use V_X as the input to stage 2 to find V_o.

$$V_o = \left(-\frac{R_f}{R_i}\right) V_X = \left(-\frac{5k}{1k}\right) 3V = -15\ volts$$