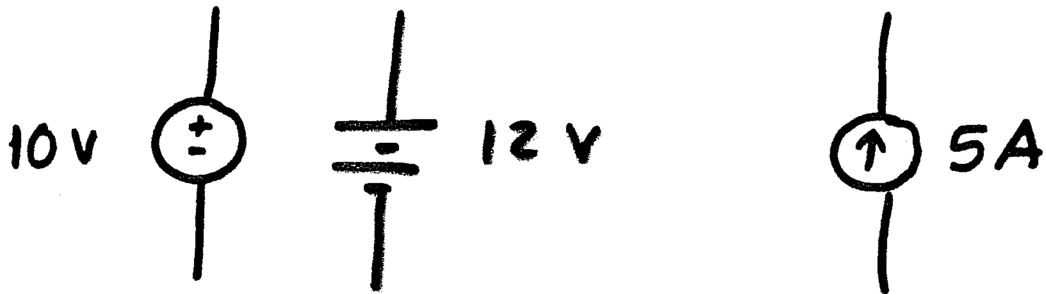
# Electrical Science

# EIT Review

## Session 1

Sources
Resistor, Ohm's Law
KVL, KCL
Node Analysis
Mesh Analysis
Power
Resistor combinations
Thevenin equivalents
Inductor
Capacitor
Transients

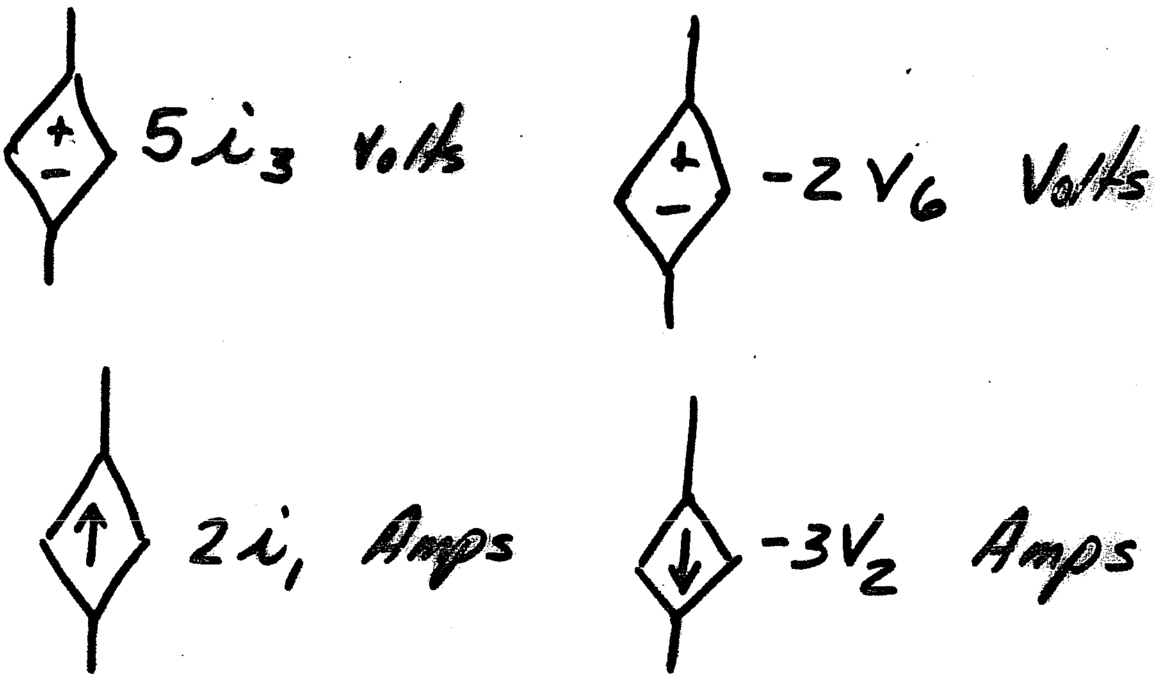
# Independent Sources




internal resistance = 0 Ω (SHORT)

internal resistance = ∞ (OPEN)

# Dependent Sources



## The Resistor


$I = V/R$ (Ohm's Law)

Assume current enters positive terminal. Then

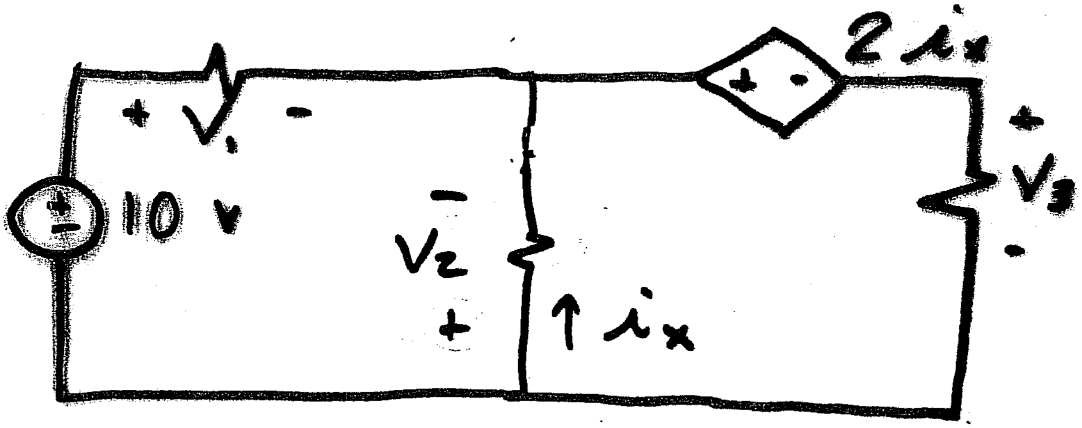
$$P_{\text{absorbed}} = V \cdot I$$

$$= \frac{V^2}{R}$$

$$= I^2 R$$

# Kirchoff's Voltage Law

$\sum$ Voltages around any closed path $= 0$



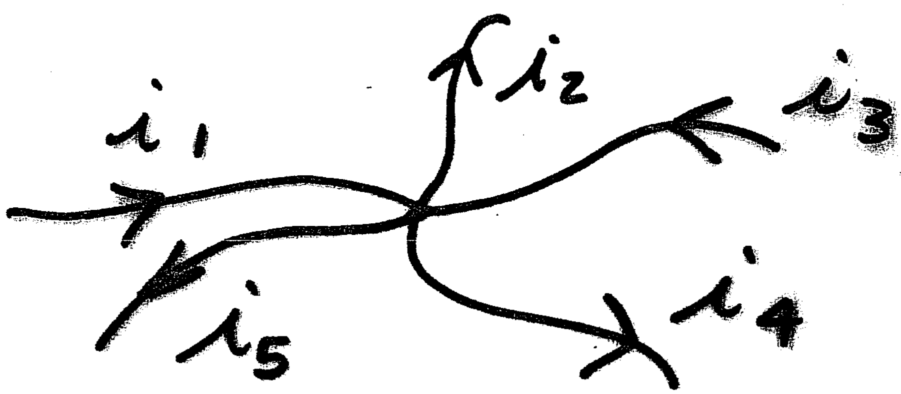

$$-10 + V_1 - V_2 = 0$$

$$-10 + V_1 + 2i_x + V_3 = 0$$

$$+V_2 + 2i_x + V_3 = 0$$

# Kirchoff's Current Law

$\sum$ currents into a node $= 0$




$$i_1 - i_2 + i_3 - i_4 - i_5 = 0$$

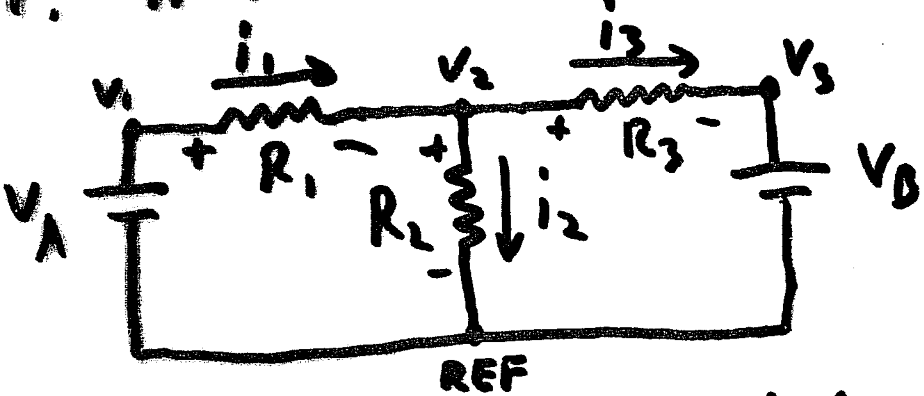
$$i_1 + i_3 = i_2 + i_4 + i_5$$

(incoming) = (leaving)

# Nodal analysis

1) Designate nodes on circuit
2) Assume currents flow to right, & downward, and label resistors' polarities accordingly
3) using KVL & KCL, write equations expressing unknown i's in terms of unknown V's S.T. # OF EQN'S = # OF UNKNOWNS. SOLVE FOR V

Ex)




BY INSPECTION: $V_1 = V_A$ & $V_3 = V_B$

$\Rightarrow$ $V_2$ is only unknown

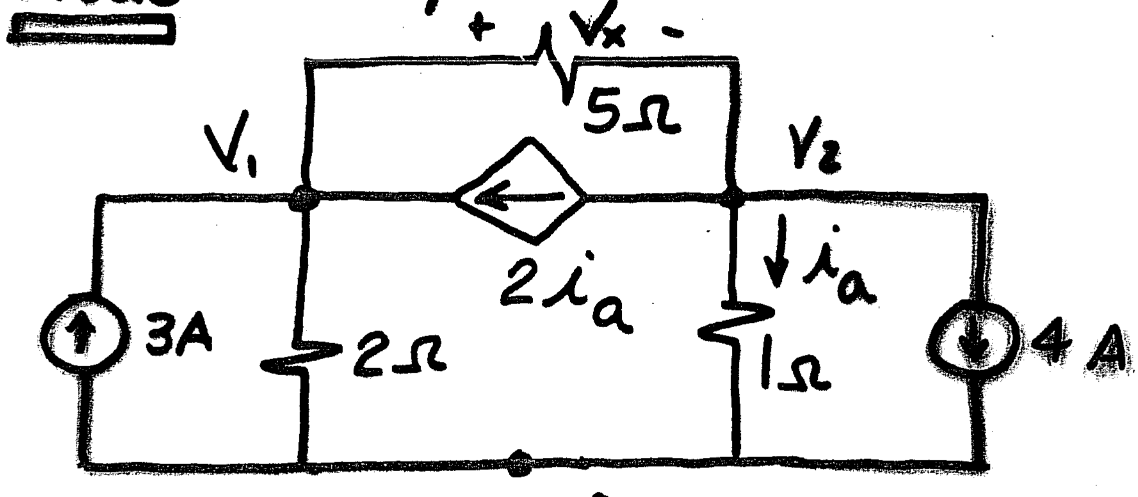
KCL: $i_{entering} = i_{leaving}$

$$\Rightarrow \frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2} + \frac{V_2 - V_3}{R_3}$$

substitute

$$\Rightarrow \frac{V_A - V_2}{R_1} = \frac{V_2}{R_2} + \frac{V_2 - V_B}{R_3}$$

# Node Analysis:




Find $V_x$. $\quad (V_x = V_1 - V_2)$

| $V_1$ | $V_2$ | Sources |
|---|---|---|
| $\left(\frac{1}{2} + \frac{1}{5}\right)$ | $-\frac{1}{5}$ | $3 + 2i_a$ |
| $-\frac{1}{5}$ | $\left(\frac{1}{5} + \frac{1}{1}\right)$ | $-2i_a - 4$ |

$$0.7\,V_1 - 0.2\,V_2 = 3 + 2i_a$$

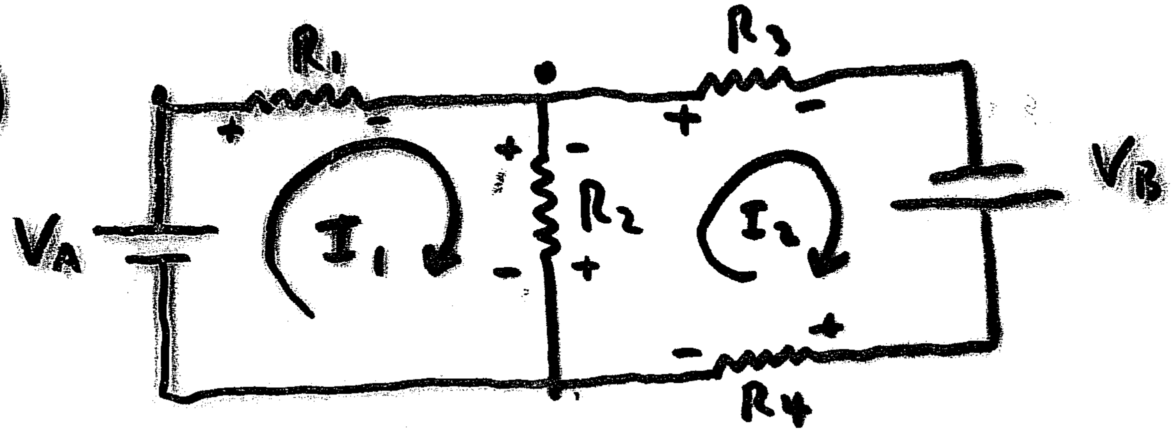
$$-0.2\,V_1 + 1.2\,V_2 = -2i_a - 4$$

$$i_a = V_2 / 1$$

# Mesh analysis

1) Designate I's around loops in a CW circular fashion.
2) Label polarities on resistors accordingly.
3) Use KVL's to write simultan. eq'ns
4) SOLVE FOR I's

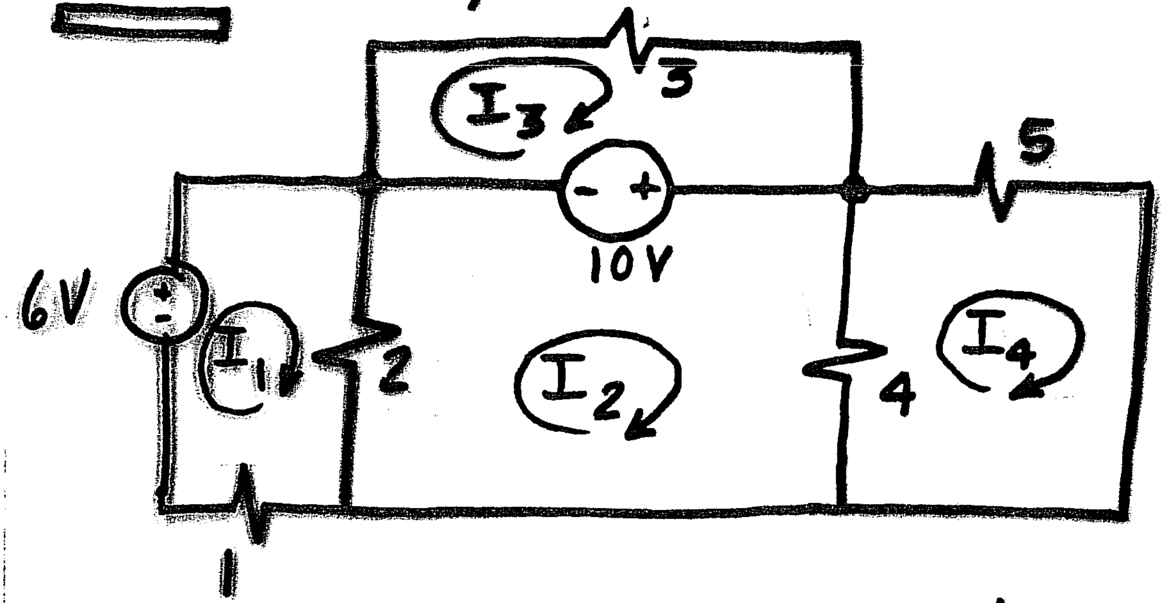
Ex)




KVL, LOOP 1: $I_1 R_1 + (I_1 - I_2) R_2 - V_A = 0$

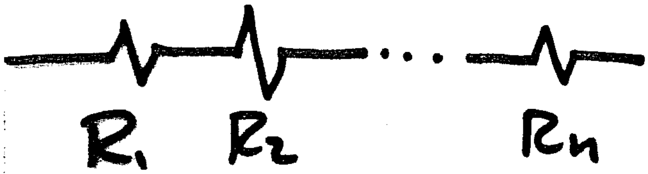
KVL, LOOP 2: $I_2 R_3 - V_B + I_2 R_4 + (I_2 - I_1) R_2 = 0$

# Mesh Analysis:

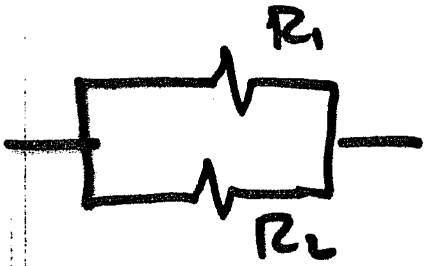



| $I_1$ | $I_2$ | $I_3$ | $I_4$ | Sources |
|---|---|---|---|---|
| 2+1 | -2 | 0 | 0 | 6 |
| -2 | 2+4 | 0 | -4 | 10 |
| 0 | 0 | 3 | 0 | -10 |
| 0 | -4 | 0 | 4+5 | 0 |

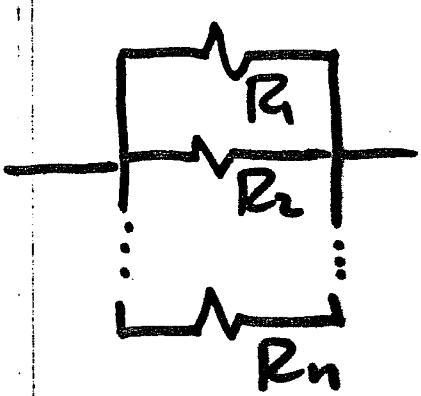
# Resistor Combinations:



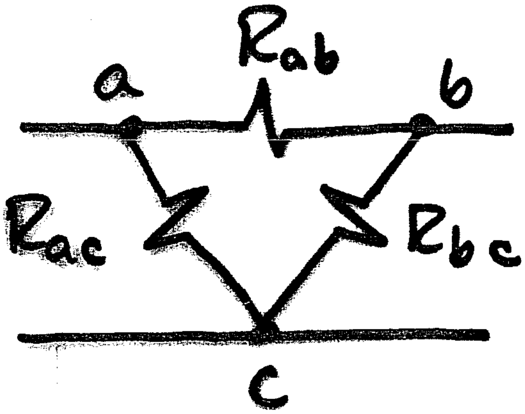

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

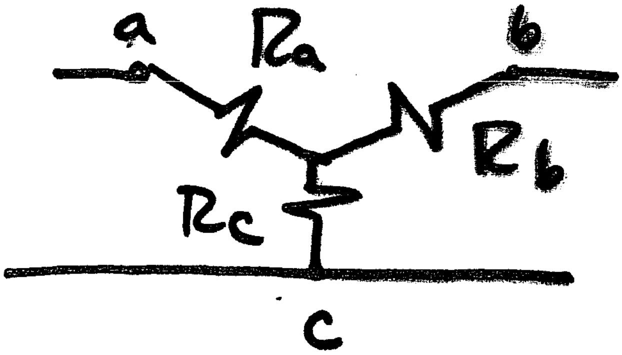



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$




$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



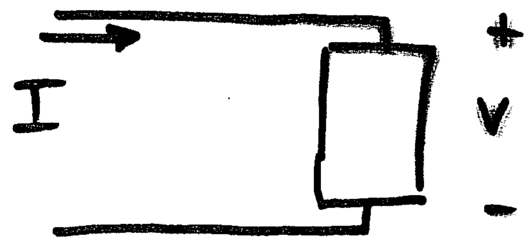




$$R_a = \frac{R_{ac} R_{ab}}{R_{ac} + R_{bc} + R_{ab}} \qquad R_{ac} = \frac{R_a R_c + R_b R_c + R_a R_b}{R_b}$$

$$R_x = \frac{\text{Product Adjacent}}{\Sigma}$$

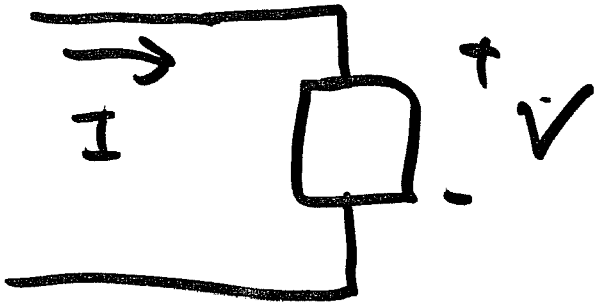
$$R_{xy} = \frac{\Sigma \text{ Pairwise Products}}{\text{Opposite}}$$

## Power:



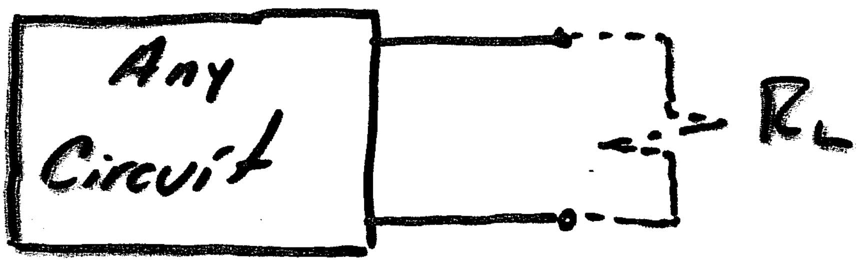

$$P_{\text{absorbed}} = V I$$

If element is a source delivering energy to the rest of the circuit, $I$ (or $V$) will be negative, and $P_{\text{absorbed}}$ will be negative.

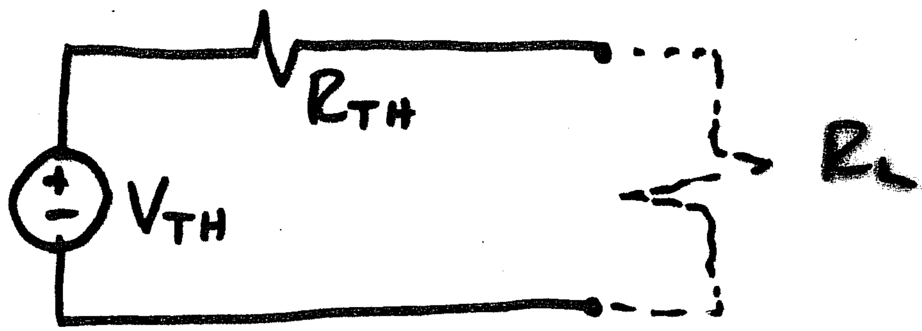



| I | V | POWER |
|---|---|---|
| + | + | ABSORBED |
| + | − | DELIVERED |
| − | + | DELIVERED |
| − | − | ABSORBED |

# Thevenin Equivalent:




$\equiv$




$V_{TH}$ = Calculate open circuit V

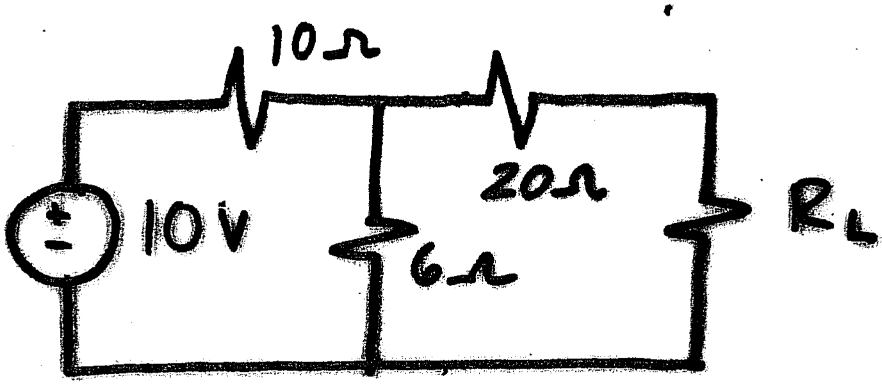
$R_{TH}$ = Turn off all sources ($V=0, I=0$) & measure R

(This works only if all sources are independent)

If dependent sources are present,

$$R_{TH} = \frac{V_{\text{open circuit}}}{I_{\text{short circuit}}}$$

Ex:

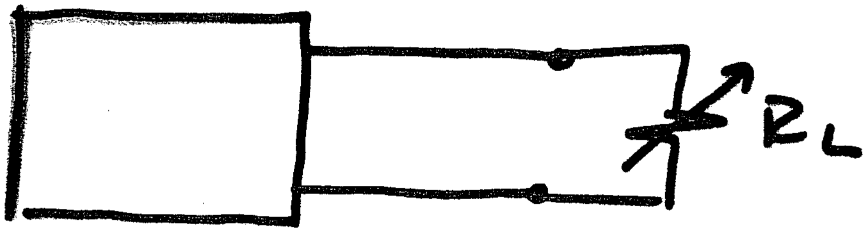



$$V_{TH} = \left(\frac{10}{10+6}\right)\cdot 6 = 3.75\text{ V}$$

$$R_{TH} = 20 + \frac{6\cdot 10}{6+10}$$

$$= \underline{23.75\ \Omega}$$

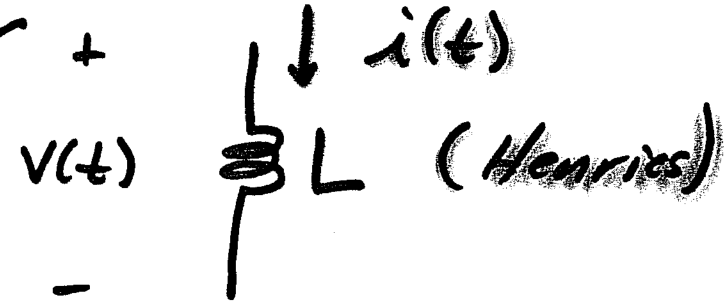
# Maximum Power Transfer




Maximum power is delivered to $R_L$ if $R_L$ is set equal to $R_{TH}$.

For example of previous page, set $R_L$ to $23.75\,\Omega$.

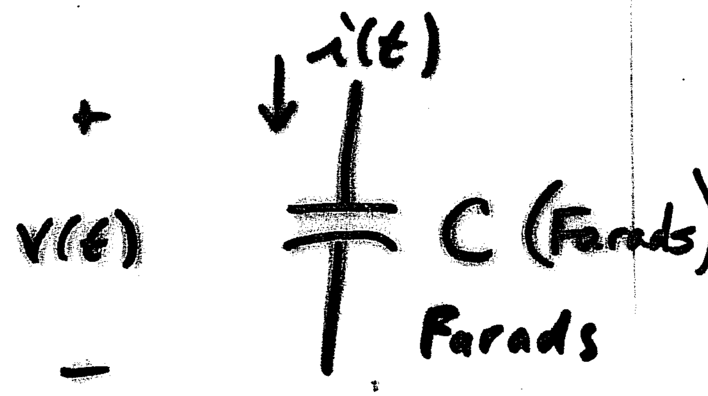
## The Inductor




$$v(t) = L\, di(t)/dt$$

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)$$

## The Capacitor




$$i(t) = C\, dv(t)/dt$$

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0)$$

## RL & RC Transients

For a network with R's, constant V and/or I sources, and one L or one C, the solution for any voltage or current is of the form:

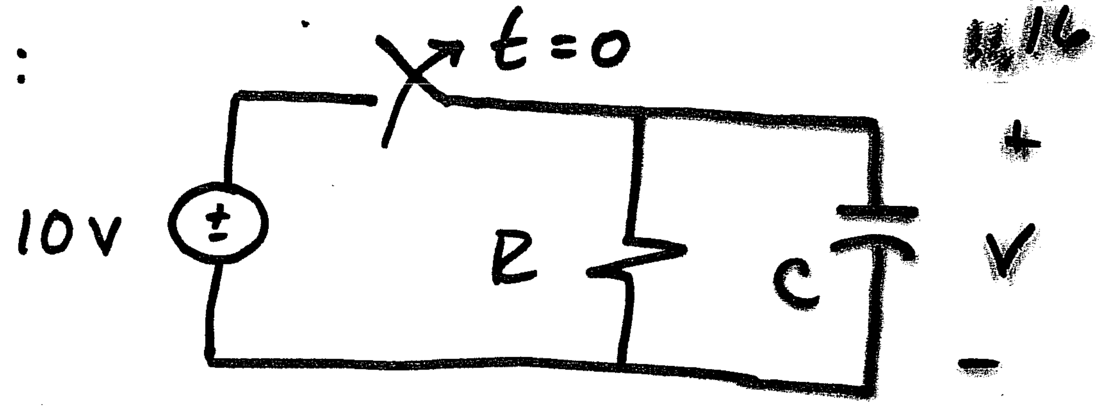
$$x = A + Be^{-t/\tau}$$

where $\tau = R_{eq} C$ (cap ckt)

or $\tau = L / R_{eq}$ (ind ckt)

∴ $R_{eq}$ is resistance "seen" by L or C (Thevanin's resistance)

Ex:



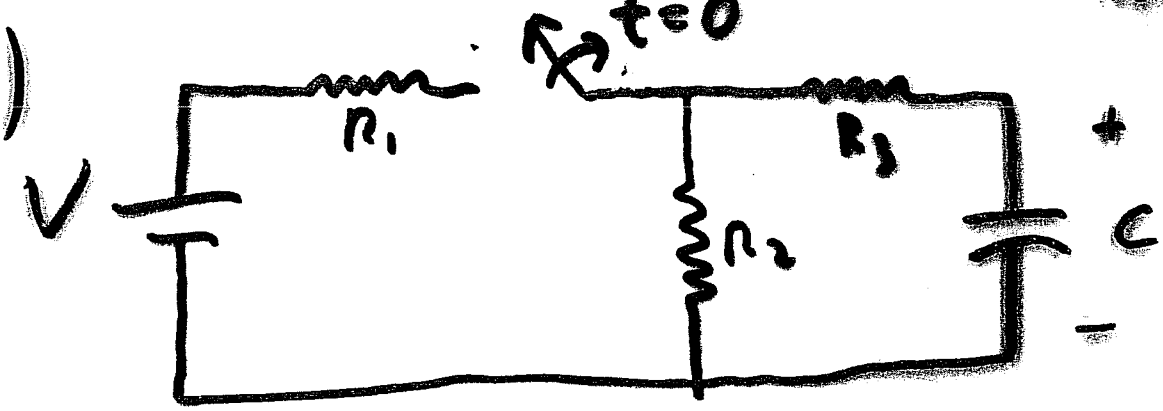

$$v(t) = A + Be^{-t/RC}$$

$$v(0^-) = 10 = A + Be^{0} = A + B$$

$$v(\infty) = 0 = A$$

$$\therefore \quad v(t) = 10e^{-t/RC}$$

Ex)




$V_c(0^-) = \dfrac{V\,(R_2)}{R_1 + R_2}$ $\qquad \tau = C(R_1+R_2)$

$$V_c(t) = V_c(0^-)\, e^{-t/C(R_1+R_2)}$$

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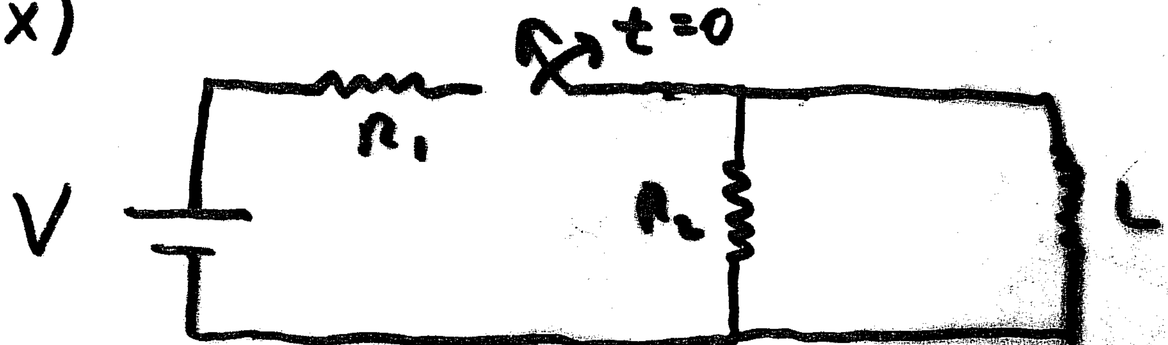
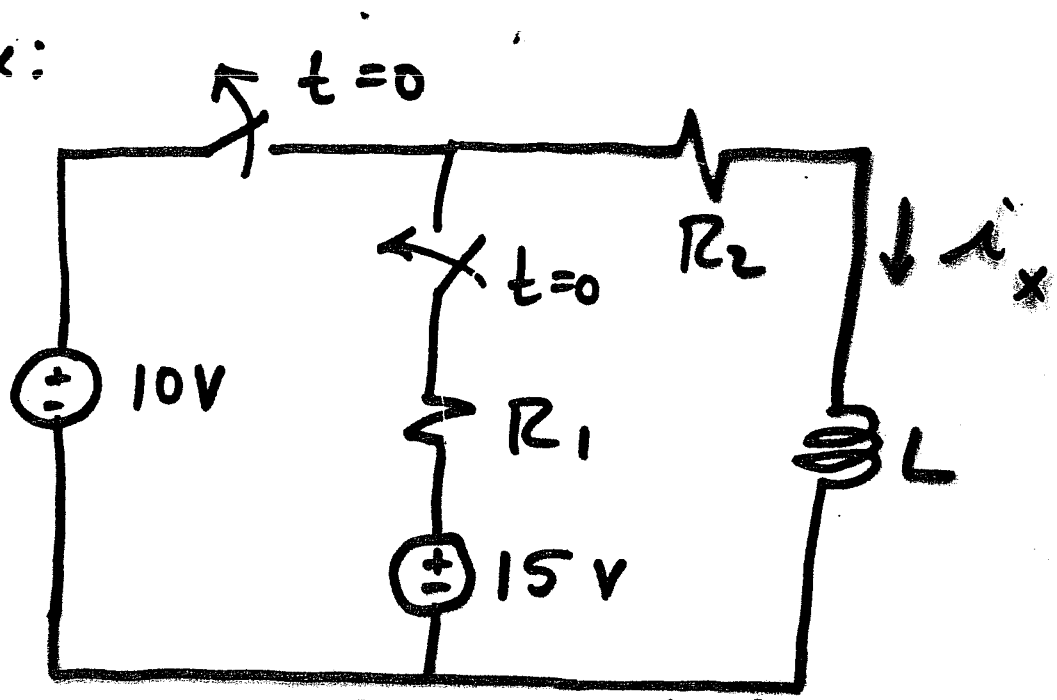
Ex)




$i_L(0^-) = \dfrac{V}{R_1}$ (L ACTS LIKE <u>SHORT</u>)

$\Rightarrow I_L(t) = i_L(0^-)\, e^{-t/\tau}$ ; $\tau = \dfrac{L}{R_2}$

Ex:




$$i_x(t) = A + Be^{-t/\tau}$$

$$i_x(0-) = \cancel{A+}B = 10/R_2$$

$$i_x(\infty) = A = 15/(R_1+R_2)$$

$$\tau = L/(R_1+R_2)$$

Note: From $v_L = L\,di/dt$ & $i_c = C\,dv/dt$, $v_C$ and $i_L$ cannot change instantaneously.

$$\therefore\ i_x(0+) = i_x(0-)$$

When currents and voltages are constant, energy stored in C's & L's is given by:

$$E_L = \frac{1}{2} L i_L^2$$

$$E_C = \frac{1}{2} C v_C^2$$