Electrical Service

EIT Review

Session 2

Phasors
Impedance
Resonance
Power & RMS Units
Complex Power
Phasors

\[ V(t) = V_m \cos(\omega t + \phi) \]

\[ \overrightarrow{V} = V_m \angle \phi \]

Units: \( \frac{\text{rad}}{s} \)
Phasors are combined using complex arithmetic.

\[ i_1 = 5 \cos (5t + 45^\circ) \]

\[ i_2 = ? = i_1 + i_3 + i_4 \text{ (by KCL)} \]

\[ i_3 = 7 \cos (5t - 135^\circ) \]

\[ i_4 = 3 \cos 5t \]
Impedance (Sinusoidal analysis only)

\[ I = \sqrt{V/Z} \quad \text{(Ohm's law)} \]

\[ \frac{1}{Z} = R \]

\[ \frac{1}{Z} = \frac{1}{j\omega C} = \frac{-j}{\omega C} \]

\[ Z = j\omega L \]

\[ \omega = 2\pi f \quad \text{Units: rad/s} \]

\[ Z = R + \frac{j\omega L}{j\omega C} \]

\[ Z \rightarrow \]

\[ \text{阻抗} \]

\[ \text{Re} \quad \text{Im} \]

\[ \frac{1}{\omega C} \]
\[ \begin{align*}
V &= 15 \angle 45^\circ \\
\bar{z} &= 3 + j(2)(2) = 3 + j4 = 5 \angle 53.1^\circ \\
\bar{I} &= \frac{\bar{V}}{\bar{z}} = \frac{15 \angle 45^\circ}{5 \angle 53.1^\circ} \\
\bar{I} &= 3 \angle -8.1^\circ \\
n(t) &= 3 \cos(2t - 8.1^\circ)
\end{align*} \]
Z's combine like R's:

\[ Z_{\text{eq}} = 5 \angle 30^\circ + 6 \angle -18^\circ \]

\[ Z_{\text{eq}} = \frac{Z_1 \cdot Z_3}{Z_1 + Z_2} \]

\[ Z_{\text{eq}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}} \]
Resonance

\[ R \quad \frac{1}{C} \quad L \]

At \( \omega \) such that \( \omega_r L = \frac{1}{\omega_r C} \), \( z_c = z_l \).

\( \omega_r \) & \( \frac{1}{\sqrt{L C}} \) cancel leaving only \( R \).

If elements are in parallel, \( z_c \) & \( z_l \) combine to form an infinite impedance, leaving only \( R \)!

At \( \omega_r = \frac{1}{\sqrt{L C}} \), \( z = R \).
Power & RMS Units

\[ p(t) = V(t) i(t) \] (instantaneous power)

Average Power = \[ \int p(t) \, dt \] over one cycle.

\[ \cdot |V| |I| \cos \theta \]

where
\[ V = \text{voltage (in RMS units)} \]
\[ I = \text{current (in RMS units)} \]
\[ \theta = \text{angle between } V \text{ and } I. \]
RMS units are defined as:

$$V_{rms} = \sqrt{T \int_0^T V^2(t) \, dt}$$

For sinusoids: $$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

For other waveforms, you must carry out the calculation.
\[ 6 \cos(3\theta + 5^\circ) \] V

Find average power delivered to \( Z \).

\[ Z = 2 \angle -15^\circ \]

\[ \overline{V} = \frac{6 \angle 5^\circ}{\sqrt{2}} \]

\[ \overline{I}_{\text{rms}} = \frac{\overline{V}_{\text{rms}}}{Z} = \frac{6\sqrt{2} \angle 5^\circ}{2 \angle -15^\circ} \]

\[ \overline{I}_{\text{rms}} = \frac{3}{\sqrt{2}} \angle 20^\circ \]

\[ P_{\text{ave}} = (\frac{6}{\sqrt{2}})(\frac{3}{\sqrt{2}}) \cos(20^\circ - 5^\circ) \]

\[ P_{\text{ave}} = \frac{18}{2} \cos(15^\circ) \]
Notes:

1. Don't worry about whether to use $+\theta$ or $-\theta$, since $\cos(\theta) = \cos(-\theta)$

2. $\cos \theta$ is called the Power Factor.

3. If you are asked for the power at any given time, you must use $p(t) = v(t)i(t)$. 
Ex:  For previous problem, find power being delivered to Z at $t = 0.1$ sec.

\[ V(t) = 6 \cos(3t + 5^\circ) \]
\[ i(t) = 3 \cos(3t + 20^\circ) \]
\[ P(t) = 18 \cos(3t + 5^\circ) \cos(3t + 20^\circ) \]
\[ P(t) = 18 \cos(\frac{3t}{3} + 5^\circ) \cos(\frac{3t}{3} + 20^\circ) \]

Careful!!!

[Note: Radians vs. Degrees]
Complex Power

\[ P = |V||I| \cos \theta \quad \text{Real Power} \]

\[ Q = |V||I| \sin \theta \quad \text{Reactive Power} \]

\[ S = VI^* \quad \text{Complex Power} \]

Units:
- \( P \) = watts
- \( Q \) = vars (volts-amperes reactive)
- \( S \) = VA (volts-amperes)
If the $\phi$ of $Z$ is positive (inductive), the current lags the voltage, and the power factor is described as lagging. The standard problem is:

$$\frac{V}{I} = Z_c$$

How much $Z_c$ is needed to make $I$ and $V$ in phase (1.0 power factor)?
Ex:

A single phase inductive load takes 50 kVA at 0.6 p.f. lagging. The amount of capacitance (in kilovars) needed to improve the power factor to unity is:

Sol'n:

\[ Q_c = 5 \sin \theta \]

\[ \theta = \cos^{-1}(0.6) = 53.1^\circ \]

\[ Q_c = (50 \text{ kVA}) \sin (53.1^\circ) \]

\[ Q_c = 40 \text{ kVARs} \]