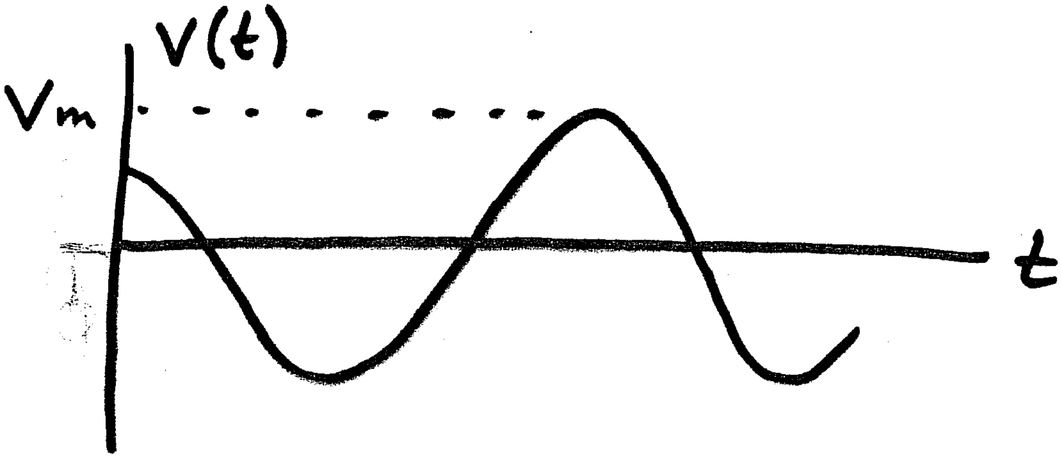
Electrical Science

EIT Review

Session 2

Phasors
Impedance
Resonance
Power & RMS units
Complex Power

Phasors

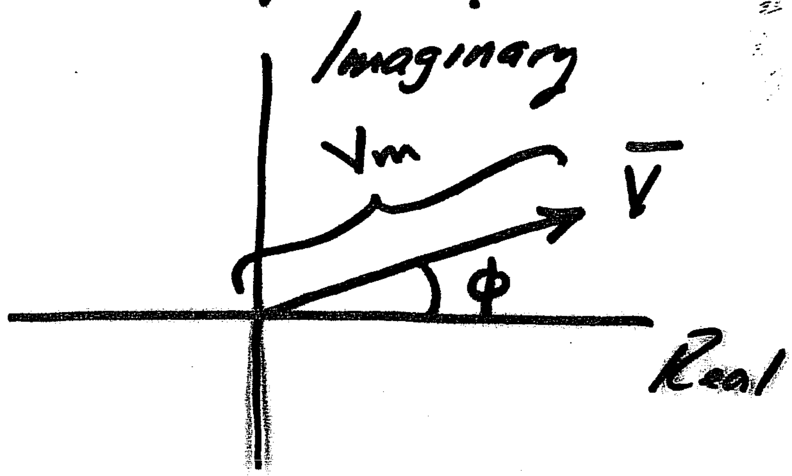


$$V(t) = V_m \cos(\omega t + \phi)$$

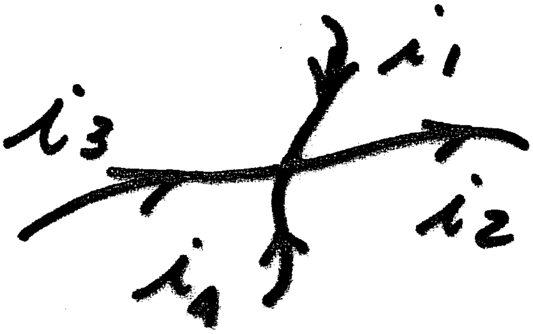
$\omega \triangleq 2\pi f$

UNITS: $\frac{RAD}{s}$

$$\overline{V} = V_m \angle \phi$$



Phasors are combined using complex arithmetic.

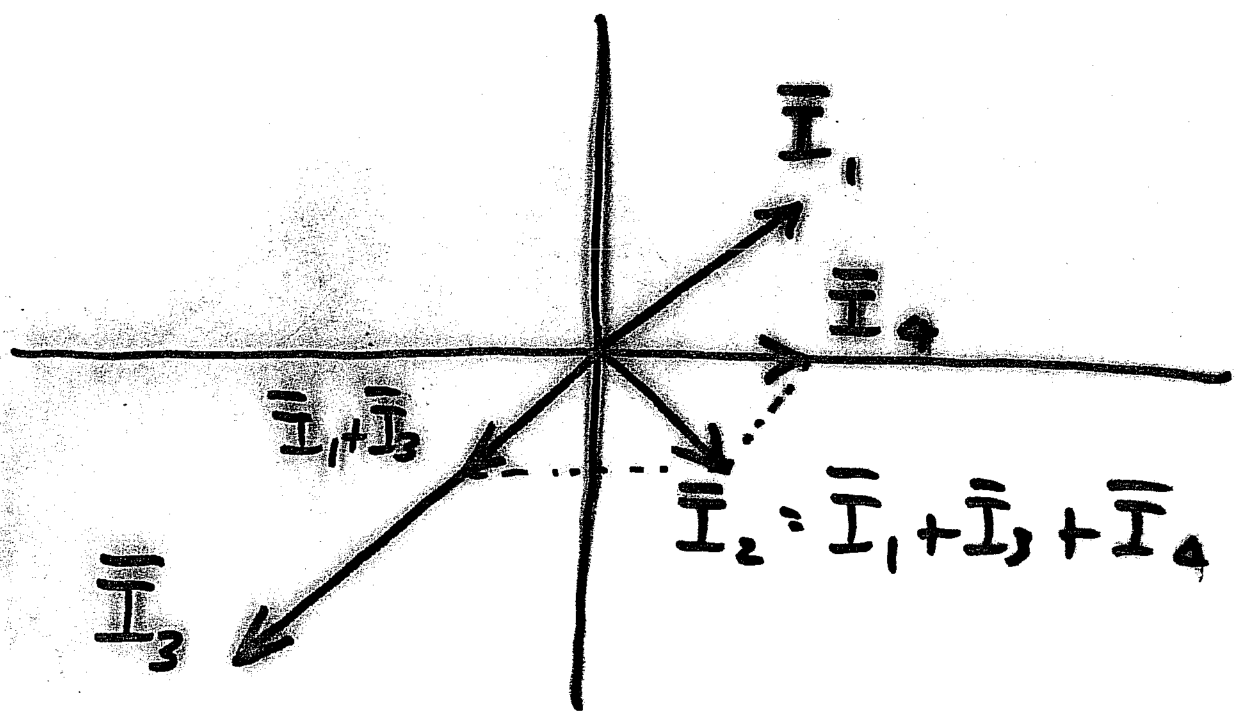


$i_1 = 5\cos(5t + 45°)$

$i_2 = ? = i_1 + i_3 + i_4$ (by KCL)

$i_3 = 7\cos(5t - 135°)$

$i_4 = 3\cos 5t$



Impedance (Sinusoidal analysis only)

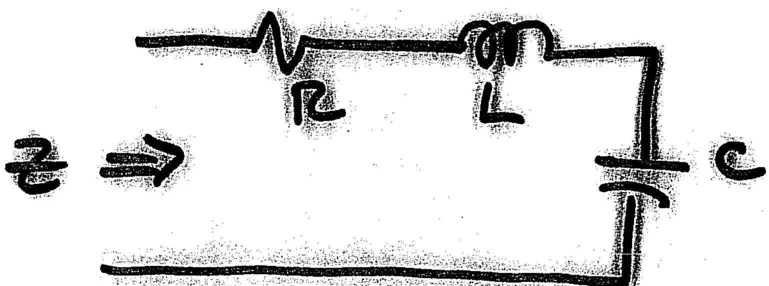
$\bar{I} = \bar{V}/Z$ (Ohm's law)

 $Z = R$

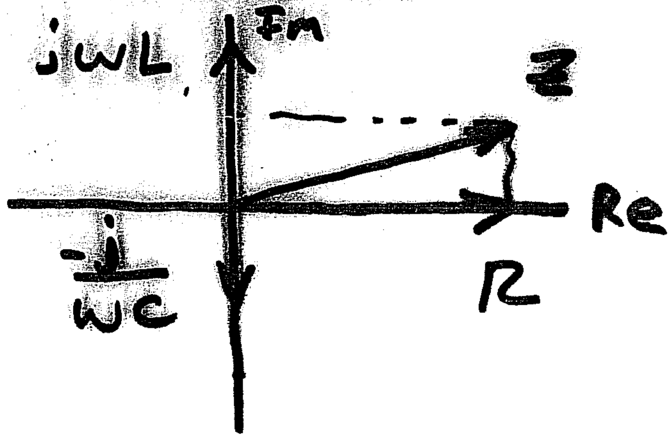
 $Z = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

 $Z = j\omega L$

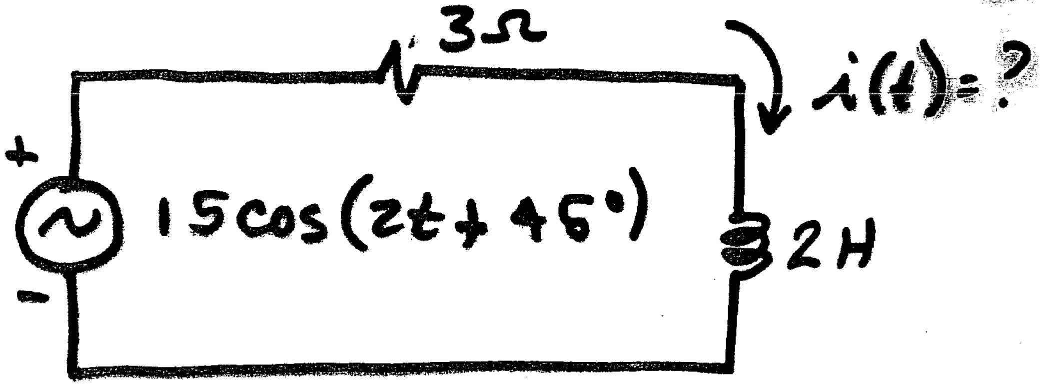
$\omega \triangleq 2\pi f$
UNITS: $\frac{RAD}{S}$



$Z = R + j\omega L + \frac{1}{j\omega C}$



Ex:



$$\bar{V} = 15\angle 45^\circ$$

$$Z = 3 + j(2)(2) = 3 + j4 = 5\angle 53.1^\circ$$

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{15\angle 45^\circ}{5\angle 53.1^\circ}$$

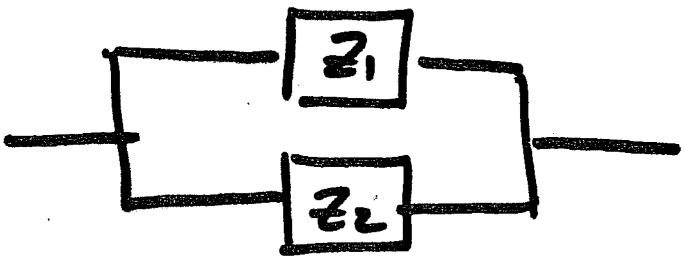
$$\bar{I} = 3\angle -8.1^\circ$$

$$i(t) = 3\cos(2t - 8.1^\circ)$$

Z's combine like R's:

$5\angle 30^\circ$ $6\angle -18^\circ$

$$Z_{eq} = 5\angle 30^\circ + 6\angle -18^\circ$$

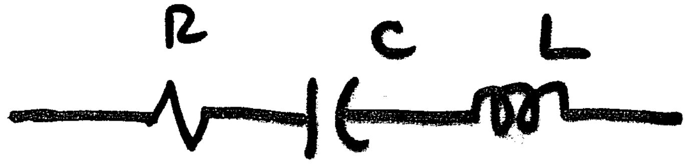


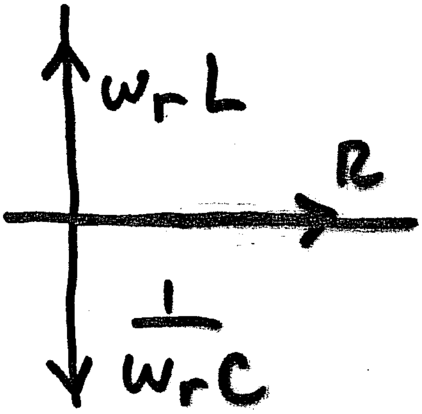
$$Z_{eq} = \frac{Z_1 Z_2}{\underbrace{Z_1 + Z_2}_{\text{lots of complex arithmetic}}}$$

$$\Updownarrow$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}$$

Resonance



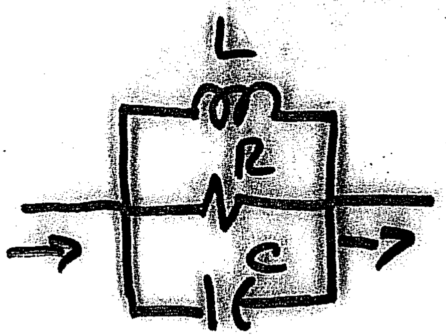


At ω such that $\omega_r L = \frac{1}{\omega_r C}$, $Z_L = Z_C$

C & L Z's cancel leaving only R.

$$\boxed{\omega_r = 1/\sqrt{LC}}$$

If elements are in parallel, Z_L & Z_C combine to form an <u>infinite</u> impedance, leaving only R!



At $\omega_r = 1/\sqrt{LC}$,

$Z = R$.

Power & RMS Units

$p(t) = v(t)\,i(t)$ (instantaneous power)

Average Power $= \int p(t)\,dt$ over one cycle

$= |\bar{V}||\bar{I}|\cos\theta$

where

$\bar{V}$ = voltage (in rms units)

$\bar{I}$ = current (in rms units)

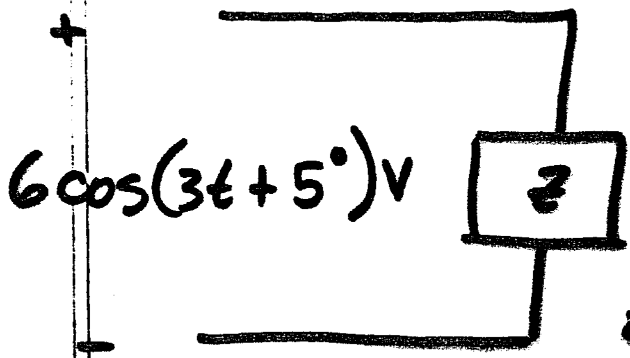
θ = $\measuredangle$ between $\bar{V}$ & $\bar{I}$.

RMS units are defined as:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

For <u>sinusoids</u>: $V_{rms} = \dfrac{V_{max}}{\sqrt{2}}$

For other waveforms, you must carry out the calculation.



Find average power delivered to Z.

$Z = 2\angle -15^\circ$

$$\bar{V}_{rms} = \frac{6}{\sqrt{2}}\angle 5^\circ$$

$$\bar{I}_{rms} = \frac{\bar{V}_{rms}}{Z} = \frac{6/\sqrt{2}\ \angle 5^\circ}{2\ \angle -15^\circ}$$

$$\bar{I}_{rms} = \frac{3}{\sqrt{2}}\angle 20^\circ$$

$$P_{ave} = \left(\frac{6}{\sqrt{2}}\right)\left(\frac{3}{\sqrt{2}}\right)\cos(20^\circ - 5^\circ)$$

$$P_{ave} = \frac{18}{2}\cos(15^\circ)$$

Notes:

① Don't worry about whether to use + or − θ, since $\cos(\theta) = \cos(-\theta)$

② $\cos\theta$ is called the Power Factor.

③ If you are asked for the power at any given time, you must use $p(t) = v(t)\,i(t)$.

Ex: For previous problem, find power being delivered to Z at $t = 0.1$ sec.

$$v(t) = 6\cos(3t + 5^\circ)$$

$$i(t) = 3\cos(3t + 20^\circ)$$

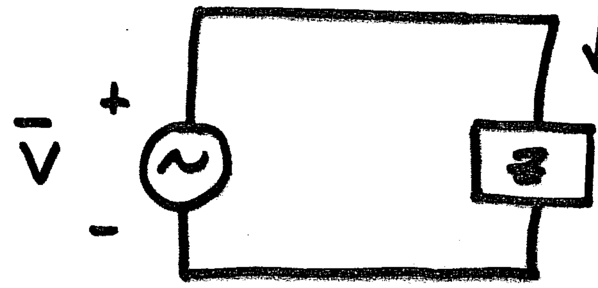
$$p(t) = 18\cos(3t + 5^\circ)\cos(3t + 20^\circ)$$

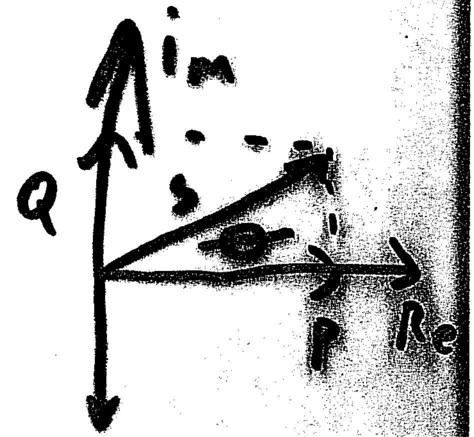
$$p(t) = 18\cos(.3 + 5^\circ)\cos(.3 + 20^\circ)$$

<u>**Careful!!**</u>

radians degrees

Complex Power





$P = |\bar{V}||\bar{I}| \cos\theta$ (Real Power)

$Q = |\bar{V}||\bar{I}| \sin\theta$ (Reactive Power)

$S = V I^*$ (Complex Power)

$*$ => COMPLEX CONJUGATE

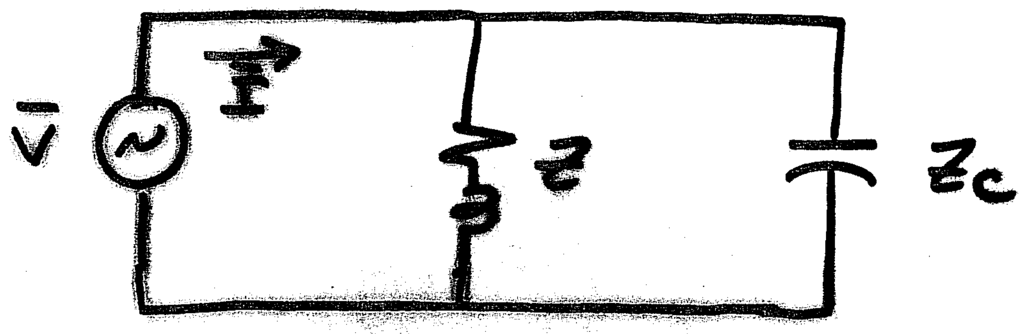
Units:

P - watts

Q - VARs (volt-amperes reactive)

S - VA (volt-amperes)

If the $\measuredangle$ of Z is positive (inductive) the current lags the voltage, and the power factor is described as lagging. The standard problem is :

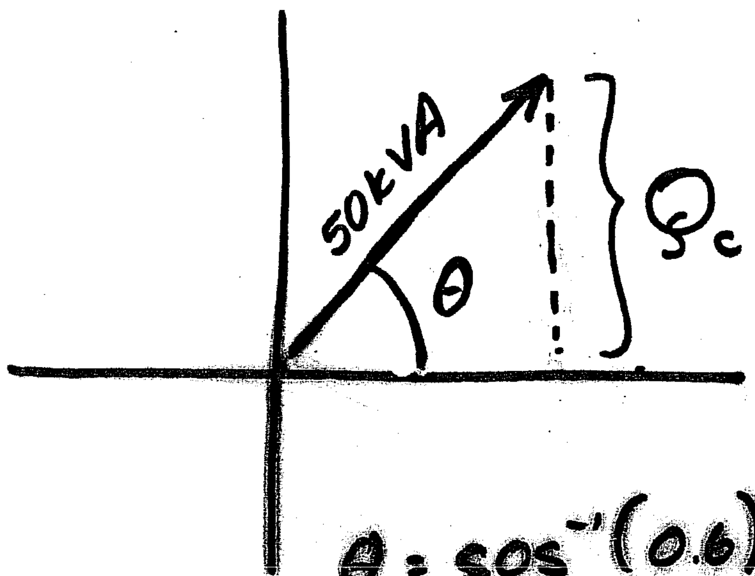


How much Z_c is needed to make $\bar{I}$ and $\bar{V}$ in phase (1.0 power factor)?

Ex:

A single phase inductive load takes 50 kVA at 0.6 p.f. lagging. The amount of capacitance (in kilovars) needed to improve the power factor to unity is:

Sol'n:



$\theta = \cos^{-1}(0.6)$

$= 53.1°$

$Q_c = S \sin\theta$

$Q_c = (50\text{ kVA}) \sin(53.1°)$

$\boxed{Q_c = 40 \text{ kVARs}}$