3.2-1

For yielding of the gross section,

\[ A_g = 7(3/8) = 2.625 \text{ in.}^2, \quad P_n = F_y A_g = 36(2.625) = 94.5 \text{ kips} \]

For fracture of the net section,

\[ A_e = (3/8) \left[ 7 - \left( 1 + \frac{1}{8} \right) \right] = 2.203 \text{ in.}^2 \]

\[ P_n = F_u A_e = 58(2.203) = 127.8 \text{ kips} \]

a) The design strength based on yielding is

\[ \phi_r P_n = 0.90(94.5) = 85.05 \text{ kips} \]

The design strength based on fracture is

\[ \phi_f P_n = 0.75(127.8) = 95.85 \text{ kips} \]

The design strength for LRFD is the smaller value: \( \phi_f P_n = 85.1 \text{ kips} \)

b) The allowable strength based on yielding is

\[ \frac{P_n}{\Omega_i} = \frac{94.5}{1.67} = 56.59 \text{ kips} \]

The allowable strength based on fracture is

\[ \frac{P_n}{\Omega_i} = \frac{127.8}{2.00} = 63.9 \text{ kips} \]

The allowable service load is the smaller value: \( P_n/\Omega_i = 56.6 \text{ kips} \)

Alternate solution using allowable stress: For yielding,

\[ F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi} \]

and the allowable load is \( F_t A_g = 21.6(2.625) = 56.7 \text{ kips} \)

For fracture,

\[ F_f = 0.5F_u = 0.5(58) = 29.0 \text{ ksi} \]

and the allowable load is \( F_f A_e = 29.0(2.203) = 63.89 = 63.89 \text{ kips} \)

The allowable service load is the smaller value = 56.7 kips
For yielding of the gross section,

\[ P_n = F_yA_g = 50(3.37) = 168.5 \text{ kips} \]

For fracture of the net section,

\[ A_n = A_g - A_{\text{holes}} = 3.37 - 0.220\left(\frac{7}{8} + \frac{1}{8}\right) \times 2 \text{ holes} = 2.930 \text{ in.}^2 \]

\[ A_e = 0.85A_n = 0.85(2.930) = 2.491 \text{ in.}^2 \]

\[ P_n = F_eA_e = 65(2.491) = 161.9 \text{ kips} \]

a) The design strength based on yielding is

\[ \phi_t P_n = 0.90(168.5) = 152 \text{ kips} \]

The design strength based on fracture is

\[ \phi_t P_n = 0.75(161.9) = 121.4 \text{ kips} \]

The design strength is the smaller value: \( \phi_t P_n = 121.4 \text{ kips} \)

Let \( P_u = \phi_t P_n \)

\[ 1.2D + 1.6(3D) = 121.4, \text{ Solution is: } \{D = 20.23\} \]

\[ P = D + L = 20.23 + 3(20.23) = 80.9 \text{ kips} \]

\[ P = 80.9 \text{ kips} \]

b) The allowable strength based on yielding is

\[ \frac{P_n}{\Omega_t} = \frac{168.5}{1.67} = 100.9 \text{ kips} \]

The allowable strength based on fracture is

\[ \frac{P_n}{\Omega_t} = \frac{161.9}{2.00} = 80.95 \text{ kips} \]

The allowable load is the smaller value = 80.95 kips

\[ P = 81.0 \text{ kips} \]

Alternate computation of allowable load using allowable stress: For yielding,

\[ F_t = 0.6F_y = 0.6(50) = 30.0 \text{ ksi} \]

and the allowable load is

\[ F_tA_g = 30.0(3.37) = 101.1 \text{ kips} \]

For fracture,

\[ F_t = 0.5F_u = 0.5(65) = 32.5 \text{ ksi} \]

and the allowable load is

\[ F_tA_e = 32.5(2.491) = 80.96 \text{ kips} \]
Gross section: \[ P_n = F_y A_g = 36(5.86) = 211.0 \text{ kips} \]

Net section: \[ A_n = 5.86 - \left( \frac{5}{8} \right) \left( 1 + \frac{1}{8} \right) (2) = 4.454 \text{ in.}^2 \]

\[ U = 1 - \frac{\bar{x}}{\bar{t}} = 1 - \frac{1.03}{(3 + 3 + 3)} = 0.8856 \]

\[ A_e = A_n U = 4.454(0.8856) = 3.944 \text{ in.}^2 \]

\[ P_n = F_u A_e = 58(3.944) = 228.8 \text{ kips} \]

(a) The design strength based on yielding is \[ \phi_i P_n = 0.90(211.0) = 190 \text{ kips} \]

The design strength based on fracture is \[ \phi_i P_n = 0.75(228.8) = 172 \text{ kips} \]

The design strength is the smaller value: \[ \phi_i P_n = 172 \text{ kips} \]

Load combination 2 controls: \[ P_u = 1.2D + 1.6L = 1.2(50) + 1.6(100) = 220 \text{ kips} \]

Since \( P_u > \phi_i P_n; \) (220 kips > 172 kips), \[ \text{The member is not adequate.} \]

(b) For the gross section, The allowable strength is \[ \frac{P_n}{\Omega_i} = \frac{211.0}{1.67} = 126 \text{ kips} \]

For the net section, the allowable strength is \[ \frac{P_n}{\Omega_i} = \frac{228.8}{2.00} = 114 \text{ kips} \]
The allowable strength based on fracture is

\[
\frac{P_n}{\Omega_t} = \frac{241.5}{2.00} = 120.8 \text{ kips}
\]

The allowable load is the smaller value = 120.8 kips

\[P = 121 \text{ kips}\]

Alternate computation of allowable load using allowable stress: For yielding,

\[F_t = 0.6F_y = 0.6(50) = 30.0 \text{ ksi}\]

and the allowable load is

\[F_tA_g = 30.0(4.75) = 142.5 \text{ kips}\]

For fracture,

\[F_t = 0.5F_u = 0.5(70) = 35 \text{ ksi}\]

and the allowable load is

\[F_tA_e = 35(3.45) = 120.8 \text{ kips}\]
Compute the strength of one plate, then double it.

Gross section: \( A_g = 10(1/2) = 5.0 \text{ in.}^2 \)

Net section: Hole diameter = \( \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \text{ in.} \)

Possibilities for net area:

\[
A_n = A_g - \sum t \times (d \text{ or } d') = 5 - (1/2)(7/8)(2) = 4.125 \text{ in.}^2
\]

or \( A_n = 5 - (1/2)(7/8) - (1/2)\left[ \frac{7}{8} - \left( \frac{5}{4(6)} \right) \right] = 4.646 \text{ in.}^2 \)

Because of load transfer, use \( A_n = \frac{10}{9} \times 4.646 = 5.162 \text{ in.}^2 \) for this possibility.

or \( A_n = 5 - (1/2)(7/8) - (1/2)\left[ \frac{7}{8} - \left( \frac{2}{4(3)} \right) \right] - (1/2)\left[ \frac{7}{8} - \left( \frac{2}{4(3)} \right) \right] = 4.021 \text{ in.}^2 \)

Because of load transfer, use \( A_n = \frac{10}{8} \times 4.021 = 5.026 \text{ in.}^2 \) for this possibility.

The smallest value controls. Use \( A_n = 4.125 \text{ in.}^2 \)

\[
A_e = A_nU = 4.125(1.0) = 4.125 \text{ in.}^2
\]

\[
P_n = F\mu A_e = 58(4.125) = 239.3 \text{ kips}
\]

For two plates, \( P_n = 2(239.3) = 478.6 \text{ kips} \)

The nominal strength based on the net section is \( P_n = 479 \text{ kips} \)
Gross section nominal strength:

\[ P_n = F_y A_g = 50(3.60) = 180.0 \text{ kips} \]

Net section nominal strength:

\[ A_n = 3.60 - 0.314(7/8)(2) = 3.051 \text{ in.}^2 \]
\[ U = 1 - \frac{\bar{x}}{l} = 1 - \frac{0.525}{(3 + 3)} = 0.9125 \]
\[ A_e = A_n U = 3.051(0.9125) = 2.784 \text{ in.}^2 \]
\[ P_n = F_u A_e = 65(2.784) = 181.0 \text{ kips} \]

Block shear strength of tension member:

The shear areas are

\[ A_{gv} = 0.314(1.5 + 3 + 3) \times 2 = 4.710 \text{ in.}^2 \]
\[ A_{nv} = 0.314[1.5 + 3 + 3 - 2.5(7/8)] \times 2 = 3.336 \text{ in.}^2 \]

The tension area is

\[ A_{nt} = 0.314[3.0 - (0.5 + 0.5)(7/8)] = 0.6673 \text{ in.}^2 \]

For this type of connection, \( U_{bs} = 1.0 \), and from AISC Equation J4-5,

\[ R_n = 0.6F_u A_{nv} + U_{bs}F_u A_{nt} \]
\[ = 0.6(65)(3.336) + 1.0(65)(0.6673) = 173.5 \text{ kips} \]

with an upper limit of

\[ 0.6F_y A_{gv} + U_{bs}F_u A_{nt} = 0.6(50)(4.710) + 1.0(65)(0.6673) = 184.7 \text{ kips} \]
The nominal block shear strength of the tension member is therefore 173.5 kips.

Block shear strength of gusset plate:

\[ A_{gv} = \frac{3}{8} (1.5 + 3 + 3) \times 2 = 5.625 \text{ in.}^2 \]

\[ A_{nv} = \frac{3}{8} [1.5 + 3 + 3 - 2.5(7/8)] \times 2 = 3.984 \text{ in.}^2 \]

\[ A_{nt} = \frac{3}{8} [3 - (0.5 + 0.5)(7/8)] = 0.7969 \text{ in.}^2 \]

From AISC Equation J4-5,

\[ R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt} \]

\[ = 0.6(58)(3.984) + 1.0(58)(0.7969) = 184.9 \text{ kips} \]

with an upper limit of

\[ 0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(5.625) + 1.0(58)(0.7969) = 167.7 \text{ kips} \]

The nominal block shear strength of the gusset plate is therefore 167.7 kips. The gusset plate controls, and the nominal block shear strength of the connection is 167.7 kips.

(a) Design strength for LRFD:

For tension on the gross area, \( \phi_t P_n = 0.90(180.0) = 162 \text{ kips} \)

For tension on the net area, \( \phi_t P_n = 0.75(181.0) = 136 \text{ kips} \)

For block shear, \( \phi R_n = 0.75(167.7) = 126 \text{ kips} \)

Block shear controls. \[ \text{Maximum factored load = design strength = 126 kips} \]

(b) Allowable strength for ASD:

For tension on the gross area, \( \frac{P_n}{\Omega_t} = \frac{180.0}{1.67} = 108 \text{ kips} \)

For tension on the net area, \( \frac{P_n}{\Omega_t} = \frac{181.0}{2.00} = 90.5 \text{ kips} \)

For block shear, \( \frac{R_n}{\xi_c} = \frac{167.7}{2.00} = 83.9 \text{ kips} \)

Block shear controls. \[ \text{Maximum service load = allowable strength = 83.9 kips} \]
Gross section: \( A_g = 5.87 \text{ in.}^2, \quad P_n = F_y A_g = 50(5.87) = 293.5 \text{ kips} \)

Net section: \( \text{Hole diameter} = 1 \frac{1}{8} + \frac{1}{8} = 1.25 \text{ in.} \)

\[
A_n = A_g - \sum t_w \times (d \text{ or } d') = 5.87 - 0.448(1.25) = 5.310 \text{ in.}^2
\]

or \[
A_n = 5.87 - 0.448(1.25) - 0.448 \left[ 1.25 - \frac{(1.5)^2}{4(4)} \right] = 4.813 \text{ in.}^2
\]

\[
U = 1 - \frac{x}{l} = 1 - \frac{0.583}{6(1.5)} = 0.9352
\]

\[
A_e = A_n U = 4.813(0.9352) = 4.501 \text{ in.}^2
\]

\[
P_n = F_u A_e = 70(4.501) = 315.1 \text{ kips}
\]

(a) Gross section: \( \phi_i P_n = 0.90(293.5) = 264 \text{ kips} \)

Net section: \( \phi_i P_n = 0.75(315.1) = 236 \text{ kips (controls)} \)

\[
P_u = 1.2D + 1.6L = 1.2(36) + 1.6(110) = 219 \text{ kips} < 236 \text{ kips} \quad \text{(OK)}
\]

Since \( P_u < \phi_i P_n \) (219 kips < 236 kips), \( \text{The member has enough strength.} \)

(b) Gross section: \[
\frac{P_n}{\Omega_i} = \frac{293.5}{1.67} = 176 \text{ kips}
\]

Net section: \[
\frac{P_n}{\Omega_i} = \frac{315.1}{2.00} = 158 \text{ kips (controls)}
\]

\[
P_a = D + L = 36 + 110 = 146 \text{ kips} < 158 \text{ kips} \quad \text{(OK)}
\]

Since \( P_a < \frac{P_n}{\Omega_i} \) (146 kips < 158 kips), \( \text{The member has enough strength.} \)