CHAPTER 6 - BEAM-COLUMNS

6.2-1

(a) LRFD solution:

From the column load tables, the compressive design strength of a W12 × 106 with \( F_y = 50 \) ksi and \( K_p L = 1.0 \times 14 = 14 \) feet is

\[
\phi_c P_n = 1130 \text{ kips}
\]

From the design charts in Part 3 of the Manual, for \( L_b = 14 \) ft and \( C_b = 1.0 \),

\[ \phi_b M_n = 597 \text{ ft-kips} \quad (\text{Since the bending moment is uniform, } C_b = 1.0.) \]

The factored axial compressive load is

\[ P_a = 1.2P_D + 1.6P_L = 1.2(0.25 \times 250) + 1.6(0.75 \times 250) = 375.0 \text{ kips} \]

The factored bending moment is

\[ M_a = 1.2M_D + 1.6M_L = 1.2(0.25 \times 240) + 1.6(0.75 \times 240) = 360.0 \text{ ft-kips} \]

Determine which interaction equation controls:

\[
\frac{P_a}{\phi_c P_n} = \frac{375}{1130} = 0.3319 > 0.2 \quad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}
\]

\[
\frac{P_a}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ax}}{\phi_b M_{nx}} + \frac{M_{ay}}{\phi_b M_{ny}} \right) = \frac{375}{1130} + \frac{8}{9} \left( \frac{360}{597} + 0 \right) = 0.868 < 1.0 \quad (\text{OK})
\]

This member satisfies the AISC Specification

(b) ASD solution:

From the column load tables, the allowable compressive strength of a W12 × 106 with \( F_y = 50 \) ksi and \( K_p L = 1.0 \times 14 = 14 \) feet is

\[ \frac{P_n}{\Omega_c} = 754 \text{ kips} \]

From the design charts in Part 3 of the Manual, for \( L_b = 14 \) ft and \( C_b = 1.0 \),

\[ \frac{M_n}{\Omega_b} = 398 \text{ ft-kips} \quad (\text{Since the bending moment is uniform, } C_b = 1.0.) \]

The total axial compressive load is \( P_a = 250 \) kips

Determine which interaction equations controls:

\[
\frac{P_a}{P_n/\Omega_c} = \frac{250}{754} = 0.3316 > 0.2 \quad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}
\]

\[
\frac{P_a}{P_n/\Omega_c} + \frac{8}{9} \left( \frac{M_{ax}}{M_{ax}/\Omega_b} + \frac{M_{ay}}{M_{ay}/\Omega_b} \right) = \frac{250}{754} + \frac{8}{9} \left( \frac{240}{398} + 0 \right) = 0.868 < 1.0 \quad (\text{OK})
\]

This member satisfies the AISC Specification

[6-1]
6.2-2

(a) Compute compressive strength (this shape is not in the column load tables).

For a W18 × 86, $A_g = 25.3$ in.$^2$, $r_y = 2.63$ in., and the shape is not slender (no footnote).

$$\frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.63} = 91.25 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(91.25)^2} = 34.37 \text{ ksi}$$

$$4.71 \sqrt{\frac{F_e}{F_y}} = 4.71 \sqrt{\frac{29,000}{50}} = 113.4$$

Since $KL/r = 91.25 < 113.4$, use AISC Eq. E3-2.

$$F_{cr} = 0.658(F_e/F_y)F_y = 0.658\left(\frac{34.37}{50}\right)(50) = 27.20 \text{ ksi}$$

$$P_n = F_{cr}A_g = 27.20(25.3) = 688.2 \text{ kips}$$

$$\phi_cP_n = 0.90(688.2) = 619.4 \text{ kips}$$

From the beam design charts in Part 5 of the Manual, for $L_b = 20$ ft, $\phi_bM_n = 552$ ft-kips for $C_b = 1$.

For this case, $C_b = 1.14$ (Figure 5.15, textbook). For $C_b = 1.14$,

$$\phi_bM_n = 1.14 \times 552 = 629.3 \text{ ft-kips} < \phi_bM_p = 698 \text{ ft-kips}$$

Factored axial load $= P_u = 1.2D + 1.6L = 1.2(10) + 1.6(20) = 44.0 \text{ kips}$

$$\frac{P_u}{\phi_cP_n} = \frac{44.0}{619.4} = 7.104 \times 10^{-2} < 0.2 \therefore \text{ use Eq. 6.4 (AISC Eq. H1-1b):}$$

$$\frac{P_u}{2\phi_cP_n} + \left(\frac{M_{ux}}{\phi_bM_{ax}} + \frac{M_{uy}}{\phi_bM_{ny}}\right) = \frac{0.07104}{2} + \left(\frac{M_{ux}}{629.3} + 0\right) = 1.0$$

$$M_{ux} = 606.9 \text{ ft-kips}$$

Let $\frac{1}{8} w_uL^2 = M_{ux}$:

$$\frac{1}{8} w_u(20)^2 = 606.9 \quad \Rightarrow \quad w_u = 12.14 \text{ kips/ft}$$

$$w_u = 1.2w_D + 1.6w_L$$

$$12.14 = 1.2(0.086) + 1.6w_L, \quad \quad w_L = 7.52 \text{ kips/ft}$$

(b) From part (a), $P_n = 688.2$ kips, $\frac{P_n}{\Omega_c} = \frac{688.2}{1.67} = 412.1 \text{ kips}$

From the beam design charts in Part 5 of the Manual, for $L_b = 20$ ft,

$$\frac{M_n}{\Omega_b} = 368 \text{ ft-kips for } C_b = 1.$$ 

For this case, $C_b = 1.14$ (Figure 5.15, textbook). For $C_b = 1.14$, 

[6-2]
\[
\frac{M_a}{\Omega_b} = 1.14 \times 368 = 419.5 \text{ ft-kips} < \frac{M_p}{\Omega_b} = 464 \text{ ft-kips}
\]

Axial load = \( P_a = D + L = 10 + 20 = 30 \text{ kips} \)

\[
\frac{P_a}{P_a/\Omega_c} = \frac{30}{412.1} = 7.280 \times 10^{-2} < 0.2
\]

.: use Equation 6.6 (AISC Equation H1-1b)

\[
\frac{P_a}{2P_0/\Omega_c} + \left( \frac{M_{ax}}{M_{ax}/\Omega_b} + \frac{M_{ay}}{M_{ay}/\Omega_b} \right) = 0.07280 + \left( \frac{M_{ax}}{419.5} + 0 \right) = 1.0
\]

\( M_{ax} = 404.2 \text{ ft-kips} \)

Let \( \frac{1}{8} w_a L^2 = M_{ax} : \quad \frac{1}{8} w_a(20)^2 = 404.2 \quad \Rightarrow \quad w_a = 8.084 \text{ kips/ft} \)

\( w_D + w_L = 0.086 + w_L = 8.084, \quad w_L = 8.00 \text{ kips/ft} \)
In the plane of bending,

\[ P_{el} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_2 L)^2} = \frac{\pi^2(29,000)(1530)}{(1.0 \times 20 \times 12)^2} = 7603 \text{ kips} \]

\[ C_m = 1.0 \]

(a) LRFD solution:
\[ P_u = 1.2P_D + 1.6P_L = 1.2(10) + 1.6(20) = 44.0 \text{ kips} \]

\[ B_1 = \frac{C_m}{1 - (\alpha P_r/P_{et})} = \frac{C_m}{1 - (1.00P_u/P_{et})} = \frac{1.0}{1 - (44.0/7603)} = 1.006 \]

\[ B_1 = 1.01 \]

(b) ASD solution: \( P_a = 10 + 20 = 30.0 \) kips

\[ B_1 = \frac{C_m}{1 - (\alpha P_r/P_{et})} = \frac{C_m}{1 - (1.60P_a/P_{et})} = \frac{1.0}{1 - (1.60)(30.0)/7603} = 1.006 \]

\[ B_1 = 1.01 \]
(a) LRFD solution:

The factored-load axial force is

\[ P_u = 1.2P_D + 1.6P_L = 1.2(0.25 \times 190) + 1.6(0.75 \times 190) = 285.0 \text{ kips} \]

The factored-load end moments are

\[ M_{\text{top}} = 1.2M_D + 1.6M_L = 1.2(0.25 \times 80) + 1.6(0.75 \times 80) = 120.0 \text{ ft-kips} \]
\[ M_{\text{bot}} = 1.2M_D + 1.6M_L = 1.2(0.25 \times 75) + 1.6(0.75 \times 75) = 112.5 \text{ ft-kips} \]

From the column load tables, for \( KL = 15 \text{ ft} \), \( \phi_c P_n = 555 \text{ kips} \)

From the beam design charts in Part 3 of the Manual, for \( L_b = 15 \text{ ft} \) and \( C_b = 1.0 \),

\[ \phi_b M_n = 257 \text{ ft-kips}, \quad \phi_b M_p = 280 \text{ ft-kips} \]

Compute \( C_b \):

![Diagram](image-url)
\[
C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C} \\
= \frac{12.5(120)}{2.5(120) + 3(61.88) + 4(3.75) + 3(54.38)} = 2.260
\]

For \(C_b = 2.260\), \(\phi_b M_n = 2.260(257) = 581\) ft-kips

Since 581 ft-kips > \(\phi_b M_p\), use \(\phi_b M_n = \phi_b M_p = 280\) ft-kips

For the axis of bending,

\[
C_m = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right) = 0.6 - 0.4 \left( \frac{112.5}{120.0} \right) = 0.225
\]

\[
P_{el} = \frac{\pi^2 EI_1}{(K_1 L)^2} = \frac{\pi^2 EI_2}{(K_2 L)^2} = \frac{\pi^2 (29,000)(341)}{(15 \times 12)^2} = 3012\text{ kips}
\]

\[
B_1 = \frac{C_m}{1 - (\alpha P_d / P_{el})} = \frac{0.225}{1 - (1.00 P_d / 3012)} = 0.2485 < 1.0 \therefore \text{ use } B_1 = 1.0
\]

\[
M_u = B_1 M_n + B_2 M_{lt} = 1.0(120.0) + 0 = 120.0\text{ ft-kips}
\]

Determine which interaction equation controls:

\[
\frac{P_u}{\phi_c P_n} = \frac{285}{355} = 0.5135 > 0.2 \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}
\]

\[
\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{nx}}{\phi_b M_{nx}} + \frac{M_{ny}}{\phi_b M_{ny}} \right) = 0.5135 + \frac{8}{9} \left( \frac{120.0}{280} + 0 \right)
\]

\[
= 0.894 < 1.0 \quad \text{(OK)}
\]

This member satisfies the AISC Specification

(b) ASD solution:

From the column load tables, for \(KL = 15\text{ ft}\), \(\frac{P_n}{\Omega_c} = 369\text{ kips}\)

From the design charts in Part 3 of the Manual, for \(L_b = 15\text{ ft}\) and \(C_b = 1.0\),

\[
\frac{M_n}{\Omega_b} = 171\text{ ft-kips and } \frac{M_p}{\Omega_b} = 186\text{ ft-kips.}
\]

Compute \(C_b:\)
\[
C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} = \frac{12.5(80)}{2.5(80) + 3(41.25) + 4(2.5) + 3(36.25)} = 2.260
\]

For \( C_b = 2.260 \),
\[
\frac{M_n}{\Omega_b} = 2.260(171) = 386 \text{ ft-kips} > \frac{M_p}{\Omega_b} \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 186 \text{ ft-kips}
\]

\( P_a = 190 \) kips, \( M_{nt} = 80 \) ft-kips

For the axis of bending,
\[
C_m = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right) = 0.6 - 0.4 \left( \frac{75}{80} \right) = 0.225
\]
\[
P_{el} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_s}{(K_2 L)^2} = \frac{\pi^2(29,000)(341)}{(15 \times 12)^2} = 3012 \text{ kips}
\]
\[
B_1 = \frac{C_m}{1 - \left( \frac{a P_a}{P_{el}} \right)} = \frac{C_m}{1 - (1.60 P_a/P_{el})} = \frac{0.225}{1 - 1.60(190/3012)} = 0.2503 < 1.0 \therefore \text{ use } B_1 = 1.0
\]
\[
M_a = B_1 M_{nt} + B_2 M_B = 1.0(80) + 0 = 80.0 \text{ ft-kips}
\]

Determine which interaction equations controls:
\[
\frac{P_a}{P_n/\Omega_c} = \frac{190}{369} = 0.514 \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}
\]
\[
\frac{P_a}{P_n/\Omega_c} + \frac{8}{9} \left( \frac{M_{ax}}{M_n/\Omega_b} + \frac{M_{ay}}{M_n/\Omega_b} \right) = 0.5419 + \frac{8}{9} \left( \frac{80.0}{186} + 0 \right) = 0.924 < 1.0 \quad \text{(OK)}
\]

This member satisfies the AISC Specification

[6-11]
Determine the axial compressive design strength. Use $K_x$ for the unbraced condition.

$$\frac{K_x L}{r_x/r_y} = \frac{1.7(14)}{2.44} = 9.754 \text{ ft} < K_x L = 14 \text{ ft}$$

From the column load tables with $KL = 14$ ft, $\phi_c P_n = 774$ kips

$$\frac{P_n}{\phi_c P_n} = \frac{400}{774} = 0.5168 > 0.2 \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a).}$$

Check the braced condition first. For the axis of bending,

$$C_m = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right) = 0.6 - 0.4 \left( \frac{24}{45} \right) = 0.3867$$

$$P_{el} = \frac{\pi^2 E I}{(K_1 L)^2} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2 (29,000)(881)}{(1.0 \times 14 \times 12)^2} = 8934 \text{ kips}$$

$$B_{1x} = \frac{C_{mv}}{1 - (\alpha P_r / P_{el})} = \frac{C_{mx}}{1 - (1.00 P_d / P_{el})} = \frac{0.3867}{1 - (400/8934)}$$

$$= 0.405 < 1.0 \therefore \text{ use } B_1 = 1.0$$

Sway condition: use

[6-27]
\[ B_2 = \frac{1}{1 - \frac{a \Sigma P_{nt}}{\Sigma P_{e2}}} = \frac{1}{1 - \frac{1.00 \Sigma P_{x}}{\Sigma P_{e2}}} = \frac{1}{1 - \frac{1.00(6000)}{40000}} = 1.176 \]

The total amplified moment at the top is

\[ M_{u,\text{top}} = B_1 M_{nt} + B_2 M_u = 1.0(45) + 1.176(40) = 92.04 \text{ ft-kips} \]

The total amplified moment at the bottom is

\[ M_{u,\text{bot}} = 1.0(24) + 1.176(95) = 135.7 \text{ ft-kips} \]

Use \( M_u = 135.7 \text{ ft-kips} \). Compute the moment strength. From the beam design charts with \( L_b = 14 \text{ ft} \),

\[ \phi_b M_n = 479 \text{ ft-kips for } C_b = 1.0 \text{ and } \phi_b M_p = 521 \text{ ft-kips} \]

Using the total amplified moment, compute \( C_b \):

\[ C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} = \frac{12.5(135.7)}{2.5(135.7) + 3(35.11) + 4(21.83) + 3(78.77)} = 2.208 \]

For \( C_b = 2.208 \), \( \phi_b M_n = 2.208(479) = 1058 \text{ ft-kips} \)

Since 1058 ft-kips > \( \phi_b M_p = \phi_b M_p = 521 \text{ ft-kips} \)

Eq. 6.3 (AISC Eq. H1-1a):

\[ \frac{P_n}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ax}}{\phi_b M_{nx}} + \frac{M_{my}}{\phi_b M_{ny}} \right) = 0.5168 + \frac{8}{9} \left( \frac{135.7}{521} + 0 \right) = 0.748 < 1.0 \text{ (OK)} \]

Member is satisfactory.