Homework 3.1 3.3

ENERGY METHODS.
(a) Basic Equations
   - Equilibrium
   - Compatibility
   - Constitutive
   To derive stiffness equations

(b) Energy Formulations simplify analysis (c)
   Easier to implement for complex systems

\[ W = \int F \cdot ds = \int \left( F_x dx + F_y dy \right) = \int F_x ds_x + \int F_y ds_y \]

FOR CONSERVATIVE SYSTEMS
Independent of Path

\[ W = F_x \Delta x + F_y \Delta y \]

No work when force travels a full circle.
why do deformable bodies deform?

Internal Forces

Because of Internal Forces

Siegel Energy

P.V.W. (Review from 320)

P.V.W Rigid Bodies.

For Equilibrium:
P_x + P_y + \text{Force}_z = E_P
P_y + P_z + \text{Force}_x = \text{E}_P

Basic Equations
For Equilibrium:
\[ P_{x1} + P_{x2} + P_{x3} = \Sigma F_x \]
\[ P_{y1} + P_{y2} + P_{y3} = \Sigma F_y \]

Basic Equations:

\[ \delta W_v = P_{x1} \delta u_x + P_{x2} \delta u_x + P_{x3} \delta u_x \]
\[ (P_{x1} + P_{x2} + P_{x3}) \delta u_x = \delta W_v \]

If I have to maintain equilibrium and since \( \delta u_x \neq 0 \) in general, I require that \( \delta W_v = 0 \)

Similarly in \( y \):
\[ \delta W_v = \Sigma F_y \delta u_y = 0 \]
\[ \delta W_v = \delta W_{v1} + \delta W_{v2} \]

If a particle is in equilibrium, then for any virtual displacement:
\[ \delta W_v = 0 \]

Q: What are the conditions so that particle is in equilibrium?
A: Apply any arbitrary
\[ \delta u_x, \delta u_y \]
\[ \delta W_v = \Sigma F_x \delta u_x + \Sigma F_y \delta u_y = 0 \]
\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \text{arbitrary} \]
CONVERSE of P.V.W

A particle is in equilibrium if $\Delta W = 0$ for every independent displacement.
EXAMPLE

3 del.:

Apply virtual displacements in all 3 DOF and compute $\delta W$.

3. Apply vertical direction.

$$\delta W = V_1 \delta V_1 + V_2 \delta V_2 - F \delta V_3$$

$$\delta V_3 = \frac{1}{2} \delta V_1 + \frac{1}{2} \delta V_2$$

$$\left[ V_1 - F(1 - \varepsilon) \right] \delta V_1 + \left[ V_2 - F(\varepsilon) \right] \delta V_2 = 0$$

To satisfy $\delta W = 0$

$$V_1 - F(1 - \varepsilon) = 0$$

$$V_2 - F(\varepsilon) = 0$$

Horizontal:

$$H_i \delta A_i = 0 \Rightarrow H_i = 0$$

$$\delta W = H_1 \delta A_1 = 0 \Rightarrow H_1 = 0$$

$$V_3 - F \varepsilon = 0$$

$$\delta M_1 = 0$$

$$\delta M_2 = 0$$

$$V_2 - L - F \varepsilon = 0$$

$$V_2 - F \varepsilon = 0$$