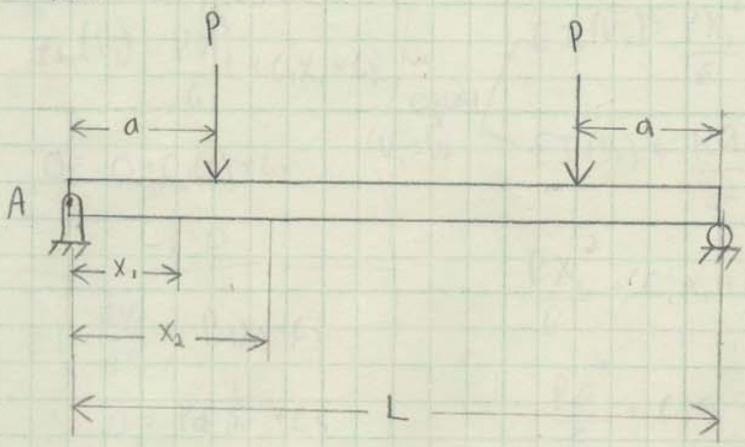
Given:



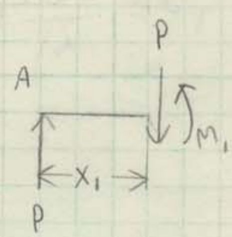
100

*EI is constant

Find: Equations of the elastic curve for the beam using x_1 and x_2 coordinates. Specify slope at A and max deflection

Solution:

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \rightarrow M(x) = EI\frac{d^2v}{dx^2}$$



$\Sigma M_A = 0 = -Px_1 + M_1$

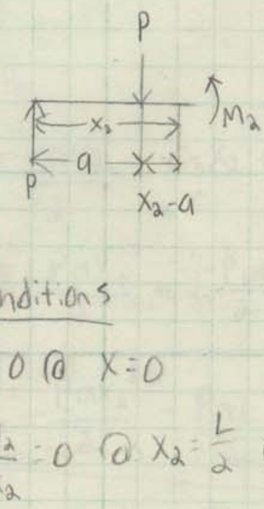
$M_1 = Px_1$

$M_1 = Px_1$

$EI\frac{d^2v_1}{dx_1^2} = Px_1$ with respect to x

$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$

* $EI(v_1) = \frac{Px_1^3}{6} + C_1x_1 + C_2$



$\Sigma M_A = 0 = -Pa + M_2$

$M_2 = Pa$

$M_2 = Pa$

$EI\frac{d^2v_2}{dx_2^2} = Pa$

$EI\frac{dv_2}{dx_2} = Pax_2 + C_3$

* $EI(v_2) = \frac{Pax_2^2}{2} + C_3x_2 + C_4$

Conditions

$v_1 = 0$ @ $x = 0$

$\frac{dv_2}{dx_2} = 0$ @ $x_2 = \frac{L}{2}$ (due to symmetry)

$v_1 = v_2$ (if $x_1 = x_2 = a$)

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ (if $x_1 = x_2 = a$)

Problem 8-4

From boundary conditions:

$$EI(v_1) = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$0 = 0 + C_1(0) + C_2$$

$$C_2 = 0$$

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3$$

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = -\frac{PaL}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

equal

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3$$

$$\frac{Pa^2}{2} + C_1 = Pa^2 + -\frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

$$EI(v_1) = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

equal $v_1 = v_2$

$$EI(v_2) = \frac{Pax_2^2}{2} + C_3x_2 + C_4$$

$$\frac{Px_1^2}{6} + C_1x_1 + \cancel{C_2}^{\,0} = \frac{Pax_2^2}{2} + C_3x_2 + C_4$$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} + -\frac{PaL(a)}{2} + C_4$$

$$* \quad C_1a - C_4 = \frac{Pa^3}{3} - \frac{Pa^2L}{2}$$

$$\left(\frac{Pa^2}{2} - \frac{PaL}{2}\right)a - C_4 = \frac{Pa^3}{3} - \frac{Pa^2L}{2}$$

$$\frac{Pa^3}{2} - \cancel{\frac{Pa^2L}{2}} - \frac{Pa^3}{3} + \cancel{\frac{Pa^2L}{2}} = C_4$$

$$C_4 = \frac{Pa^3}{6}$$

$$M_1 = Px_1$$

$$EIv_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$EIv_1 = \frac{Px_1^3}{6} + \left(\frac{Pa^2 - PaL}{2}\right)x_1 + 0$$

$$\boxed{= \frac{Px_1^3}{6} + \frac{Pax_1(a-L)}{2}}$$

$$M_2 = Pa$$

$$EIv_2 = \frac{Pax_2^2}{2} + C_3x_2 + C_4$$

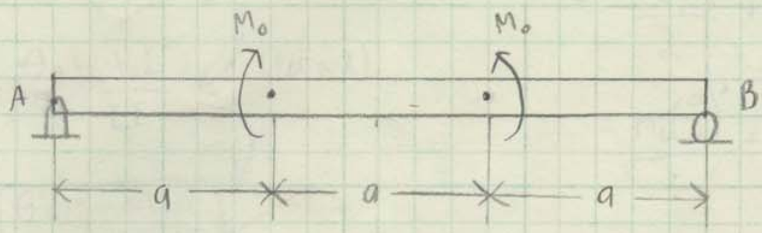
$$EIv_2 = \frac{Pax_2^2}{2} + \left(-\frac{PaL}{2}\right)x_2 + \frac{Pa^3}{6}$$

$$\boxed{= \frac{Pax_2^2}{2} - \frac{PaL\,x_2}{2} + \frac{Pa^3}{6}}$$

Problem 8-6

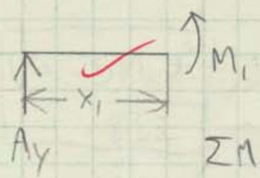
Given:

$*\ EI$ is constant



Find: Maximum deflection of beam and slope at A using double integration method

Solution:



$\Sigma M_A = 0 = M_1$

$M_1 = 0$

$M_1 = 0$

$EI \dfrac{d^2 v_1}{dx_1^2} = 0$

$EI \dfrac{dv_1}{dx} = C_1$

$EI\, v_1 = C_1 x_1 + C_2$

At $x_1 = 0,\ v_1 = 0$

$0 = 0 + C_2$

$C_2 = 0$

$M_2 = M_0$

$EI \dfrac{d^2 v_2}{dx_2^2} = M_0$

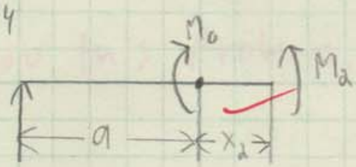
$EI \dfrac{dv_2}{dx_2} = M_0 x_2 + C_3$

$EI\, v_2 = \dfrac{M_0 x_2^2}{2} + C_3 x_2 + C_4$

At $x_2 = \dfrac{a}{2},\ \dfrac{dv_2}{dx_2} = 0$

$0 = \dfrac{M_0 a}{2} + C_3$

$C_3 = \dfrac{-M_0 a}{2}$



$\Sigma M_A = 0 = -M_0 + M_2$

$M_2 = M_0$

Conditions

At $x_1 = a,\ x_2 = 0$

$v_1 = v_2$

$\dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dv_2}$

$C_1 x_1 + \cancel{C_2}^{\,0} = \cancel{\dfrac{M_0 x_2^2}{2}}^{\,0} + \cancel{C_3 x_2}^{\,0} + C_4$

$C_1 a = C_4$

$C_1 = \dfrac{-M_0 a}{2}, \quad C_4 = \dfrac{-M_0 a^2}{2}$

At $X_1 = 0$

$$EI \frac{dv_1}{dx_1} = -\frac{M_o a}{2}$$

$$\theta_A = \frac{M_o L^2}{EI} \quad (\text{for } x = L)$$

$$\boxed{\theta_A = \frac{-M_o a}{2EI}}$$

At $X_2 = \frac{a}{2}$

$$EIV_2 = \frac{M_o X_2^2}{2} + C_3 X_2 + C_4$$

$$= \frac{M_o\left(\frac{a}{2}\right)^2}{2} + \frac{-M_o a}{2}\left(\frac{a}{2}\right) + \frac{-M_o a^2}{2}$$

$$= \frac{M_o a^2}{8} - \left(\frac{M_o a^2}{4}\right)^{2} - \left(\frac{M_o a^2}{2}\right)^{4}$$

(for x=L)

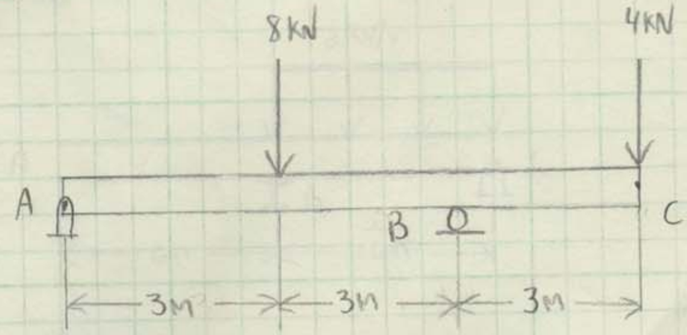
$$V = \frac{M_o L^2}{2EI} \leftarrow \frac{M_o a^2}{8} - 2\left(\frac{M_o a^2}{4}\right) - 4\left(\frac{M_o a^2}{2}\right)$$

$$\boxed{V = \frac{-5M_o a^2}{8EI}}$$

Good job, you are one of the few people who understand how to do this problem.

Problem 8-31

~~Given:~~

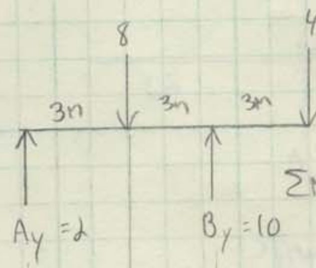


$E = 200\ GPa$

$I = 70(10^6)\ mm^4$

Find: Use conjugate beam method to determine slope and displacement at the end of C

Solution:



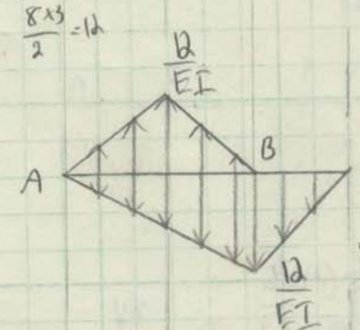
$\Sigma M_A = 0 = -(8 \times 3) + (B_y \times 6) + -(4 \times 9)$

$B_y = 10kN$

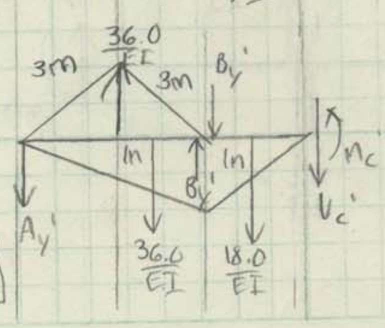
$\Sigma F_y = 0 = -8 - 4 + 10 + A_y$

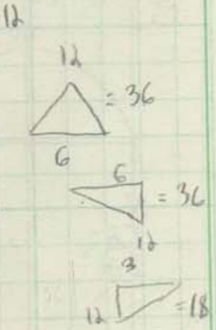
$A_y = 2kN$

$\frac{8 \times 3}{2} = 12$



$\frac{4 \times 6}{2} = 12$





$$\Sigma M_A' = 0 = \frac{36.0}{EI}(1) + {}^{-}B_y(6)$$

$$B_y = \frac{6.0}{EI}$$

$$\Sigma F_y' = 0 = -\frac{6.0}{EI} + \frac{18.0}{EI} - V_c'$$

$$V_c' = \frac{-24}{EI}$$

$$V_c' = \theta_B = \frac{-24 \times 10^3}{(200 \times 10^9)(70 \times 10^{-6})}$$

$$\boxed{\theta_B = -0.001714\ \text{rads}}$$

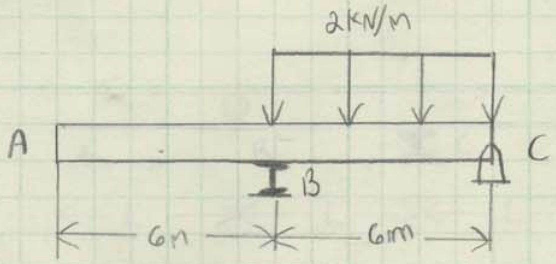
$$\Sigma M_c' = 0 = \frac{18}{EI}(2) + \frac{6}{EI}(3) + M_c'$$

$$M_c' = \frac{54}{EI}$$

$$\Delta_c = M_c' = \frac{54 \times 10^3}{(200 \times 10^9)(70 \times 10^{-6})} = \boxed{0.0038571 M}$$

Problem 8-34

Given:



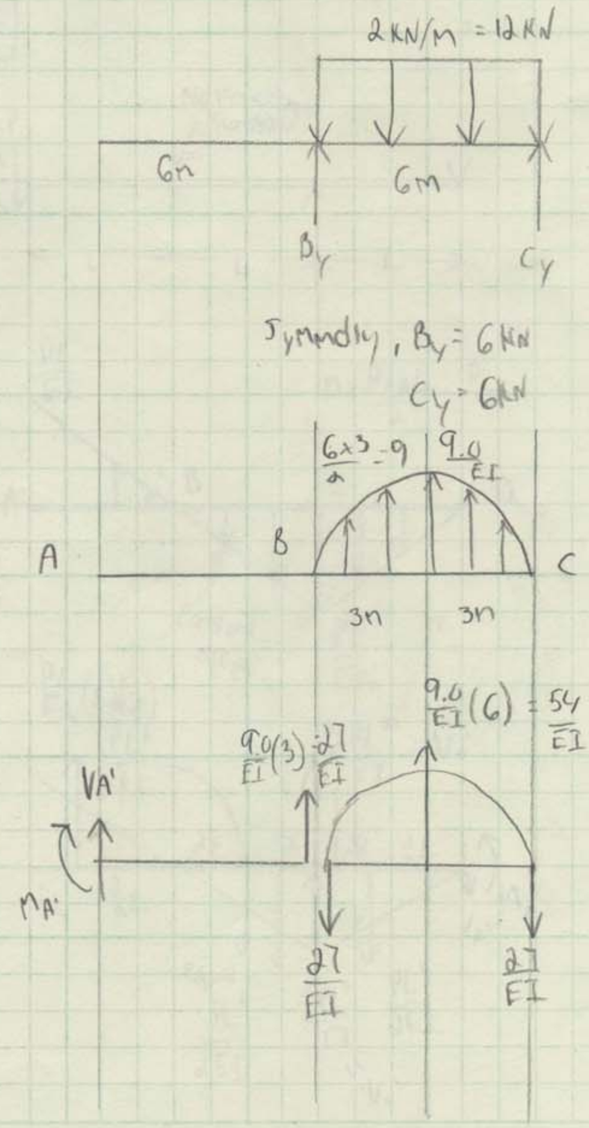
B is roller

$E = 200\ GPa$

$I = 80(10^6)\ mm^4$

Find: Use conjugate beam method to determine displacement at A

Solution:



$$\Sigma M_A' = \frac{27}{EI}(6) - M_A'$$

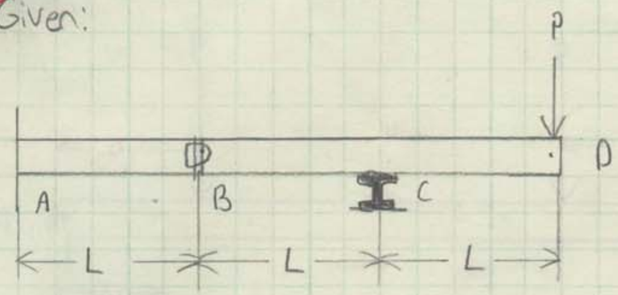
$$M_A' = \frac{162}{EI}$$

$$\Delta_A = M_A' = \frac{162}{EI}$$

$$\Delta_A = \frac{162\times10^3}{(200\times10^9)(80\times10^{-6})} = \boxed{0.010125m}$$

Problem 8-37

Given:



A is fixed support and C is roller

EI is constant

Find: Use conjugate-beam method to determine the displacement at D and slope at C

Solution

$$\boxed{\theta_c = V_c' = -\overset{+}{}\frac{2PL^2}{3EI} \quad \text{deflection is negative thus} \quad \theta_c = \frac{-2PL^2}{3EI}}$$

$$\Sigma M_{D'} = 0 = \frac{2PL^2}{3EI}(L) + \frac{PL^2}{2EI}\left(\frac{2}{3}L\right) + M_{D'}$$

$$\frac{2PL^3}{3EI} + \frac{2PL^3}{6EI} + M_{D'}$$

$$\frac{4PL^3}{6EI} + \frac{2PL^3}{6EI} + M_{D'}$$

$$\boxed{M_{D'} = -\frac{\cancel{6}PL^3}{\cancel{6}EI} = \frac{-PL^3}{EI}}$$